Empirical Implications of Efficient Market Models

ROBERT G. JAMES[1]

Abstract

Two alternative definitions of an efficient market model are developed. One is based on Bayesian probability and the other is an orthodox model. It is shown that empirical tests that reject the orthogonality property (zero correlation between a market's forecast error and information used to make the forecast) may merely reflect the Bayesian assertion that information effects probabilities. Markets may still be using all available information to form mathematical expectations of the future.

Introduction

This paper is divided into three sections. The first section contains a brief review of the difference between Bayesian and orthodox (or frequentist) probability. The second section gives two definitions of efficient market models, one based on Bayesian probability and the other based on orthodox probability with a fixed probability space. The empirical importance of differentiating between the two efficient market models is demonstrated in the third section. It is shown that regression tests that reject the orthogonality property may merely reflect the Bayesian assertion that information effects probabilities. Markets may still be using all available information to form mathematical expectations of the future. (The orthogonality property is zero correlation between a market's forecast error and information used to make the forecast.)

I. Bayesian versus orthodox probability

> All propositions are true or false, but the knowledge we have of them depends on our circumstances: and while it is often covenant to speak of propositions as certain or probable, this expresses strictly a relationship in which they stand to a corpus of knowledge, actual or hypothetical, and not a characteristic of the propositions in themselves. (Keynes, 1973, p. 4)

Keynes used the notation a/h to stress the Bayesian view that knowledge effects probability. Here, a/h is the probability of a, given the knowledge represented by h. In this

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[1] The author is Professor of Economics at California State University, Chico

notation, h contains all knowledge that is used to define the probability of a. According to Keynes and other Bayesians, probabilities are ambiguous unless information used to make the probabilities is specified. In Keynes' words,

> Clear thinking on the subject of Probability is not possible without a symbol which takes an explicit account of the premises of the argument as well as of its conclusion; and endless confusion has arisen through discussions about the probability of a conclusion without reference to the argument as a whole. I claim, therefore, the introduction of the symbol a/h as an essential step towards any progress in the subject. (Keynes, 1973, p. 119)

Although many accept the Bayesian view of probability, few use Keynes' notation. For example, Zellner (1983) summarized Keynes' view of probability as follows.

> Jeffreys, along with Keynes (1921), Uspensky (1937), Renyi (1970), and others, emphasizes that all probabilities are conditional on an initial information set, denoted by A. For example, let B represent the proposition that a six will be observed on a single flip of a coin. The degree of reasonable belief or probability that one attaches to B depends on the initial information concerning the shape and other features of the coin and the way in which it is thrown, all of which are included in the initial information set, A. Thus, the probability of B is written P(B|A), a conditional probability. The probability of B without specifying A is meaningless. (Zellner, 1983, p. 74)

A problem with Zellner's notation is that his Bayesian conditional probability is easily confused with an orthodox conditional probability. An orthodox conditional probability, usually denoted P(B|A), is calculated for a probability measure defined on a sigma-field of sets, where B and A are random variables in a fixed probability space. The probability measure defines the unconditional probability of each set in the sigma field. Thus, if P(B|A) is an orthodox conditional probability, the unconditional probability of B is defined, it is not meaningless. In short, an orthodox conditional probability necessitates the existence of unconditional probabilities. Billingsley (1986) gives a formal definition of an orthodox conditional probability and a probability space.

To avoid confusing Bayesian and orthodox conditional probabilities, in this paper, a/h denotes a Bayesian conditional probability and P(B|A) denotes an orthodox conditional probability.

The difference between a Bayesian and orthodox conditional probability can be demonstrated with a simple example. Suppose a bag contains n balls, m of which are black. The first ball drawn is black. Using orthodox conditional probability notation, the probability the second ball is black is

$$P(B|A) = (m-1) / (n-1), \qquad (1)$$

where B is the conclusion that the second ball is black and A is the knowledge that the first ball was black. Nothing about the bag or the values of m and n is reflected in A.

Using Keynes' notation

$$b/h = (m-1) / (n-1), \quad (2)$$

where b is the conclusion that the second ball is black, and h is the knowledge that the first ball was black and the knowledge that the bag originally contained n balls, m of which were black.

Efficient market models make extensive use of mathematical conditional probabilities similar to Equation (1). The confusion envisioned by Keynes is often avoided by assuming new information has no effect on probabilities, that probabilities do not “depend on, or change with, historical calendar time.” (Samuelson, 1965, p. 199) In these orthodox probability models, the future is viewed as the outcome of “...independent drawings from a fixed cumulative probability distribution function….” (Lucas and Sargent, 1981, p. xii)

Orthodox probability models can approximate some phenomena. For example, consider Samuelson’s (1965) famous simile, where a drunken sailor is likened to a mathematical random walk. The original location of the sailor and his random walk behavior defines the probabilities of all possible future locations. New sightings of the sailor are realized values of random variables in a fixed probability space–a probability space fixed by the sailor’s original location and his random walk behavior.

In contrast, suppose a drunken sailor calls on the telephone and claims to be on 14th street. Due to his drunken state there is a chance he incorrectly reported his location. A probability of one-third may be given to each of his possible locations, 13th, 14th and 15th streets. New information, the sailor’s call, defines the probability space in which expectations are made.

II. Efficient Market Models

There are two commonly accepted definitions of market efficiency; one is a general definition and the other is a narrow special case of the first. The general definition has a long and colorful history. Working used it in his analysis of how contracts are priced in futures markets. According to the general definition, “...people optimally use all the available information, including information about current policies, to forecast the future” (Mankiw, 1992, p. 310), “making the most of the information that is available to them.” (Hall and Taylor, 1993, p.24). Thus, Fama’s “...hypothesis that security prices at any point in time ‘fully reflect’ *all* available information” (Fama, 1970, p. 388) and Shiller’s definition of the efficient market model as “...all information about future prices is efficiently incorporated into today’s price...” (Shiller , 1987, p. 33) are consistent with the general definition. Using Keynes’ notation, this definition of market efficiency can be written as

$$P^{e}_{t+i} / h_t = \int_{-\infty}^{\infty} p_{t+i} \; (p_{t+i} \; / h_t) \; \partial p_{t+i} \, , \quad (3)$$

where, at time t, P^{e}_{t+i} / h_t is the market’s expectation of what the price will be at time t+i and p_{t+i} / h_t is the probability of p_{t+i}. Here, probabilities and expectations are dependent on h_t, where h_t represents all knowledge available at time t.

An alternative definition of market efficiency is

$$P^{e}_{t+i,t} = E(P_{t+i} \mid \Omega_t), \qquad (4)$$

where, at time t, $P^{e}_{t+i,t}$ is the market's expectation of what the price will be at time t+i and $E(P_{t+i,t} \mid \Omega_t)$ is the orthodox conditional expected value of P_{t+i}, given Ω_t. Here, Ω_t is the set of all random variables observed at or before time t. According to Equation (4), all probabilities are defined in a fixed probability space.

When probabilities are fixed, Equations (3) and (4) are equivalent. In both equations expectations are mathematical expectations taken over a probability density function for P_{t+i}. However, Equations (3) and (4) can have quite different empirical implications. Equation (3) uses Keynes' probability notation, where information can affect probabilities. The probability space used to calculate P^{e}_{t+i} / h_t need not be the same as the probability space used to calculate P^{e}_{t+i} / h_{t-1}. In contrast, Equation (4) is an orthodox probability model. The probability space used to calculate $P^{e}_{t+i,t}$ must be the same for all values of t and i.

Orthodox models have little if any intuitive appeal. For example, suppose a new invention is made in the year 2090. The invention doubles the world's yearly wheat production. Agents are assumed to know the probability of the invention occurring in the year 2090 and the effect of the invention on wheat prices. This is completely illogical. The invention is unknown until the year 2090; today's agents cannot know the probability and effect of an unknown invention.

Another and perhaps the more serious problem with orthodox models arises from empirical studies. In foreign exchange markets there are few transaction costs or other market imperfections. Investors are free to buy and sell freely on the basis of their expectations. However, Equation (4) is routinely rejected. Similar results have been found in other markets that appear efficient. For example, Hirsch and Lovell (1969) found that the average error in sales forecasts derived by periodically surveying a sample of reliable customers tended to be correlated with the forecast itself. Excess volatility has been found in stock prices. Others have found long-shot bias in racetrack betting and excess volatility in consumption. How should economists interpret such findings? Are forecasters simply bad forecasters? If so, why don't profit-seeking speculators correct the situation? As shown below, it may merely be that Equation (3) instead of Equation (4) is used to make expectations.

III. Empirical Implications of Efficient Market Models

Empirical implications of efficient market models depend on whether Equation (3) or (4) is used to define expectations. For example, Samuelson (1965) shows that given Equation (4), the sequence ...$P^{e}_{t,t-2}$, $P^{e}_{t,t-1}$, P_t is a martingale. A martingale is a stochastic process with well-defined properties, such as the orthogonality property. According to the orthogonality property, any forecast error is orthogonal to (i.e., uncorrelated with) information known at the time of the forecast. Thus, $P_{t+i} - P^{e}_{t+i,t}$ is uncorrelated with information known at time t. Regression tests, variance bounds tests and virtually every modern test of efficient market models use the orthogonality property as either an explicit or implicit assumption.

According to Equation (3) information can change probabilities and the sequence $...P^e_{t,t-2}$, $P^e_{t,t-1}$, P_t need not be a martingale or any other stochastic process. (A stochastic process exists in a fixed probability space.) Thus, the orthogonality property is not implied by Equation (3) and modern tests of efficient markets are not applicable. This fact is shown below using the regression test in foreign exchange markets. The analysis can be easily expanded to other tests of efficient market models.

Suppose the change in the spot rate is regressed against the forward discount, where

$$\Delta s_{t+1} = \alpha + \beta fd_t + \varepsilon_{t+1}.$$

Here, $\Delta s_{t+1} = s_{t+1} - s_t$ is the change in the spot rate; $fd_t = f_t - s_t$ is the forward discount; s_t is the spot rate at time t; f_t is the forward rate at time t, for delivery at time t+1; and β is the covariance between Δs_{t+1} and fd_t divided by the variance of fd_t. It is easy to show that, given the orthogonality property, β should not be significantly different from one. For example, the change in the spot rate can be written as

$$\begin{aligned}\Delta s_{t+1} &= s_{t+1} - f_t + f_t - s_t \\ &= s_{t+1} - f_t + fd_t.\end{aligned}$$

If the foreign exchange market is efficient, f_t equals the expected value of s_{t+1}, and

$$\Delta s_{t+1} = \Delta s^u_{t+1} + fd_t,$$

where Δs^u_{t+1} is the unexpected change in the exchange rate. According to the orthogonality property, Δs^u_{t+1} is uncorrelated with fd_t. Thus, the covariance between Δs_{t+1} and fd_t equals the variance of fd_t, and β should not be significantly different from one. Empirical estimates of β that are significantly less than one are interpreted as evidence that markets are not efficiently using available information.

In contrast, consider a case where expectations are defined by Equation (3). Specifically, suppose the only thing known about a future change in the exchange rate is that the forecast error equals

$$\varepsilon_{t+1} = \Delta s_{t+1} - \Delta s_{t+1}/h_t, \qquad (5)$$

where $\Delta s_{t+1}/h_t$ is the expected value of Δs_{t+1}, given probabilities defined by h_t. In addition, assume ε_{t+1} has a zero mean and is uncorrelated with Δs_{t+1}. The variance of $\Delta s_{t+1}/h_t$ equals the variance of Δs_{t+1} plus the variance of ε_{t+1}; and the covariance between Δs_{t+1} and $\Delta s_{t+1}/h_t$ equals the variance of Δs_{t+1}. If markets are efficient in the sense that the forward discount equals $\Delta s_{t+1}/h_t$, then β should not be significantly different from

$$VAR(\Delta s_{t+1}) / (VAR(\Delta s_{t+1}) + VAR(\varepsilon_{t+1})),$$

where VAR(...) is the variance of the term inside the parentheses. Thus, β can vary between zero and one depending on the size of $VAR(\Delta s_{t+1})$ relative to $VAR(\varepsilon_{t+1})$.

It has been argued by Lovell (1985) and others that if ε_{t+1} and Δs_{t+1} are uncorrelated, then $\Delta s_{t+1}/h_t$ is not a rational expectation of Δs_{t+1}. Rational agents will adjust their expectations using the known distributions of relevant variables. Specifically, given Equation (5), profit seeking rational agents will force the forward discount to equal

$$fd_t = \lambda_1 + \lambda_2 \Delta s_{t+1}/h_t,$$

where λ_1 is the mean of Δs_{t+1} and λ_2 is the covariance between Δs_{t+1} and $\Delta s_{t+1}/h_t$ divided by the variance of $\Delta s_{t+1}/h_t$. The new forecast error, $\Delta s_{t+1} - \lambda_1 - \lambda_2 \Delta s_{t+1}/h_t$, is orthogonal to $\Delta s_{t+1}/h_t$.

The problem with Lovell's argument is that it is not possible to calculate λ_1 and λ_2 without knowing the moments and cross-moments of Δs_{t+1} and $\Delta s_{t+1}/h_t$. When information changes probabilities, there is no reason to believe that the moments are known. In terms of the drunken sailor simile, it is not possible to know the first, second and cross-moments of the sailor's actual and reported locations from his telephone call. The call only gives his reported location and the probability space that it defines.

IV. Conclusion

Efficient market models rarely address questions of how information is synthesized. Is new information the realized value of a random variable in a fixed probability space or can information affect probabilities in which expectations are formed? Some economists downplay the importance of this question. For example, in 1984 during a conversation with Kleamer, Lucas states,

> I like to talk about distributions being known, and parameters being known, so that what one 'learns' about is the realizations of particular random variables. That's purely a question of language. But I know my language puts some people off...Milton Friedman for example. He's very influenced by Savage and by his Bayesian way of thinking about probabilities. So when I talk about people 'knowing' a probability, he just can't reach that language. (Lawson, 1988, p. 41)

Lawson wonders whether this is

> ...purely a question of language? I am suggesting that, in general, it is not; that behind these terminological differences, there is a difference in belief concerning the nature of probability. (Lawson, 1988, p. 41)

This paper adds support to Lawson's contention. If all probabilities are known, expectations follow martingales and expectations have well defined stochastic properties, such as the orthogonality property. However, if information affects probabilities, expectations neither follow martingales nor have the properties of any stochastic process. If expectations are not stochastic processes, then most tests of market efficiency are not applicable. This includes all tests that implicitly or explicitly assume orthogonality between an efficient forecast error and information known at the time of the forecast. It is surprising that this fact is not generally known. Perhaps the mathematical intricacies of stochastic processes have become so intriguing that the basis of probability and expectations have been overlooked.

Ironically, Samuelson's (1965) seminal article "Proof That Properly Anticipated Prices Fluctuate Randomly" and Malkiel's (1990) famous book *A Random Walk Down Wall Street* may be among the most damaging publications in the efficient market literature. Samuelson's article led economists to believe that expectations follow martingales while Malkiel's book led the general population to believe that asset prices can follow random walks over time. Both martingales and random walks are stochastic processes that necessitate a fixed probability space. If Bayesians are correct and information affects probabilities, neither stochastic process is useful for describing expectations or asset prices.

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