Financial decision-making and modelling under soft conditions

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Abstract

In the paper soft financial decision-making and modelling is described by fuzzy-stochastic methodology. There is not in several situations in financial decision-making to have at to disposal input data of required quality. This situation is typical for small economies and economies with weak financial market, qualitative changes. Two aspects of input data uncertainty, which are often neglected, should be distinguished; risk (stochastic) and vagueness (fuzzy). Hybrid models, fuzzy-stochastic approach is discussed and presented.

JEL classification: C6, C 44, C53, F2, F21, G1, G11, G15, G2, G21

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1. Soft financial thinking and modelling

Complexity of economic and financial systems behaviour is specified by a number of mutual interactions of particular subjects, heterogeneity of relations, multi-attribute and dynamics of the whole process. In addition, because of a man being a subject under control.

Following from a fact a human intellect (capacity and type of memory, process of recognition and thinking) does not enable all particulars to be processed in full details simultaneously, results into a need of simplification. A complex system is considered to be in some approximation limits (in certain scale). Too microscopic details are beyond the perception limits (distinguish ability). This fact is characterized as *a principle of incompatibility*, see Russel (1961), Zadeh (1965). A principle is the ability to provide accurate and simultaneously substantial assortments on its behaviour decreases directly with the system complexity and precision up to a certain sensitivity threshold. Beyond this threshold, the accuracy and substantiality eliminate each other.

Further, full - detailed accuracy in one of the elements is not useful, providing this accuracy cannot be reached in other parts of the system.

One of the vagueness reasons also is the modelling (thinking) process proceeds from the indistinguishable through gradual separating particular signs towards the distinguishable. A basis is, therefore, non-distinguish (indifference), and, therefore, indeterminacy. In addition, a man is usually not able to differentiate and designate exactly the elements and relations. This

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is connected to the implementation (realization) aspect. Even it would be possible to differentiate exactly, it is not possible to gain and process in a reasonable time all information related to complex models. This aspect is characterized as a contradiction between a conceptual accuracy and sensual vagueness.

Common economic and financial models serve as a tool for supporting decision-making in complex dynamic systems control. Ones are characterized by operating very common and abstract concepts. They comprise especially numerical concepts expressing a measure and ranking. They are followed by economic concepts generalizing properties of complex dynamic economic systems.

Economic and financial decision-making is implemented in such a way that a decision-maker always perceives the whole decision-making (vector) space in substantial correlations, finding in it a certain compact area expressing simplification and vagueness specified level of the area of decision-making.

This is realized by creating models meeting the following main aspects: Objectiveness (purposefulness), Complexity (integrity), Approximation rate, Vagueness (indeterminacy).

Based on experience, traditional means of decision-making modelling of complex systems usually do not comply with these assumptions, especially concerning the in-determination aspects and approximation rate.

2. Risk and vagueness uncertainty aspects in financial modelling and decision-making

Typical feature of financial environment is uncertainty. This term is understood mostly as risk uncertainty (probability, stochastic) and is modelled by stochastic apparatus. However, the term uncertainty has the second aspect - the vagueness (sometimes-called imprecision, non-preciseness, ambiguity) which is often neglected and could be modelled by fuzzy methodology. In this respect it is apparent that the general term uncertainty includes two aspects: risk (stochastic) and vagueness (fuzzy) ones. These terms will be used in the paper.

Distinguishing risk and vagueness confirms a discussion in financial decision-making for several years (see Keynes (1937), Ellesberg (1961), Olsen and Throuhton (2000)). What are the impacts of uncertainty on decision-making? Interesting characteristics are described in Olsen and Throuhton (2000)), here vagueness is called ambiguity. (1) Uncertainty influence selection. (2) Decision-makers are ambiguity averse in general. (3) Ambiguity causes more weight to be placed on negative information. (4) Buyers pay lower prices for and insurers require higher premium on objects or hazards subject to greater difficulty in estimation of value or probability of outcome. (5) Risk aversion and ambiguity aversion have not been seen to be highly correlated.

Thus simplification and indeterminacy identification are expressed being close to a human way of thinking, and better corresponding to a decision-making process in complex dynamic financial systems.

A decision-making modelling process itself starts with verbal terms of reference and model parameters setting for real objects and relations, too.

A result of optimisation calculations is being transferred by a reverse procedure (lexicographical approximation) to a verbal form. A decision-maker thus reaches the optimum solving in form of compact sphere of vector space.

This mentioned approach is immanently present inter alia in financial and economic modelling on complex dynamic optimisation systems.

These models usually consist of following basic parts: balance equation (cash flow balance, assets and liabilities balance), transition transformation (depreciation initiation, instalments schedules, etc.), permissible strategies limitation (cost, debts, liquidity, assets efficiency, capital appropriateness, reserve ratio, etc.), demand limitation (revenues, loans), supply limitations (investment, deposits). Global optimisation criteria are based on that of net present values.

In financial analysis and decision-making processes, all types of these parameters can be set as language variables in form (interest:"roughly 15 %", liquidity "rather 2,5", debts "roughly 45 % ", capital appropriateness "not less than 8,5 %", cost "rather 70,5 %", etc.). The same is true for relations such as "maybe less than","...surely less than", "probably less than", etc. A result of model is then a vector of control (decisive) variables (gross investment, "roughly 50 c.u.", "short-term credits "rather 30 c.u.", deposits "roughly 40 c.u." etc

Fuzzy-stochastic models

One of suitable approaches for solving the problem is to apply a fuzzy-stochastic methodology and create fuzzy-stochastic model.

There are several references combining the fuzzy and stochastic processes and hybrid fuzzy-stochastic methodology has been studied, applied and explained in several papers. We can introduce basic fuzzy-stochastic apparatus, see (e.g. Dubois and Prade (1980), Puri and Ralescu, (1986), Kruse and Meyer (1987), Kacprzyk and Fedrizzi.(1988), Viertl, (1996), Ramik and Vlach (2001), Liu and Kao (2002), Wu (2003)), fuzzy-stochastic optimisation models, see (e.g. Wang and Qiao (1993), Dubois and Prade (1980), Luhandjula (1996), Luhandjula and Gupta (1996), Chiang (2001), Sakawa and Nishizaki (2003)) and financial applications see (e.g. Inuigushi and Ramik (2000), Tanaka et. al. (2000), Zmeskal (2001), Tarrazo and Gutierrez (2001), Carlsson and Fuller (2002), Yoshida (2003)).

Financial soft hybrid modeling

It is supposed that in traditional financial methodology decision-maker attitude is exactly stated and quality (precise) data are at disposal. These assumptions are often fulfilled and stochastic risk models are sufficient and effective approach for estimation the risk. However, complexity and flexibility of financial world show that there are several situations in which described conditions (goals, criteria, and constraints) are not determined precisely (ill-structured problem) and sometimes quality-input data are not at disposal (imprecise input data or distribution functions). Such circumstances exist if financial system is non-stable, after crisis, in transition economies etc. For modelling such situations, the fuzzy-stochastic approach was developed and could be applied.

We will introduce assumptions and input parameters that could be difficult to determine precisely as real numbers or distribution functions. This aspect also implies from the decision-making process conditions. Reasons why financial methodology parameters could be understood as uncertain imply from the input data uncertainty and the approximate procedures used. There is an excellent description of limitation of stochastic financial modelling methodology in Jorion (2000), similar opinion is presented in Picoult (2000).

Portfolio, VAR and risk models do not provide a measure of absolute worst loss, only some confidence level (*risk of exceeding*).

Risk also assumes that the position is fixed over the horizon (frozen position). This assumption, however, ignores the possibility of changing the trading positions over time in response to changing market conditions (*changing position risk*).

Risk models are based on historical data and it is assumed that recent past is a good projection of future randomness. Some situations where the historical patterns change abruptly will cause havoc with models. Changing correlation coefficients can lead to drastically different measures of portfolio risk (*event and stability risk*).

Whenever there is a major change, some possibility of errors exists. This applies for instance to organisational changes, implementation new systems etc. (*transitional errors*).

All the analytical methods assume that every data are available to measure risk. A meaningful market-clearing process may not exist for some securities *(data inadequacy risk*).

Errors in model output can be due to errors in inputs. Parameters risk stems from the imprecision in parameters measurement from historical data. Some random errors are bound to happen just because of sampling variation (standard errors) and problem is often ignored. Number of observations and number of assets influence it. The bias increases as number of assets increases in relation to number of observations. The problem is also that higher number of correlations is likely to have been measured with some error probability position. The higher is correlation coefficient the more likely it will change down (*estimation risk*).

Users should realise the fundamental trade-off between using more data, which leads to more precise estimates, and focusing on more recent data, which may be safer if the risk changes over time. The problem is also that the available histories may give a distorted picture of the risk merely due to survival of the series. Since only series in existence are considered (*forecasting risk*).

Some of the risks are a direct result of the model approximation. Predictions will then inevitably produce some errors (*implementation risk*).

In the case of application a financial models the estimation of an financial data is determined by input data precision, mainly concerning of validity, quality and availability of input data. There are also problems with data frequency and stochastic error as well.

An application of fuzzy-stochastic methodology in financial modelling is being studied now. A survey of input data vagueness problems in finance and accounting decision-making is in (Siegel et al (1995), Ribeiro et al. (2000) .

We can see four basic application branches. (1) Valuation of financial instruments where basic assumptions of von Neumann and Morgenstern expected utility theory are not fulfilled, mainly assumption of subaddition. It means a probability measure is substituted and generalised by a fuzzy measure, [see for instance Simonelli (2001), Young and Zariphopoulu (2000)]. Cherubini et.al.(2001), Simonelli, (2001), Yoshida (2003). (2) Forecasting financial characteristics by robust fuzzy methodology or by neuro-fuzzy network see for example Ribeiro et al. (2000), Tseng et al. (2001). (3) Multiple criteria evaluation of corporate financial level or creditworthiness with soft aspects and applying fuzzy aggregate operators in comparing with traditional expected utility criterion, Choquet integral, Sugeno integral or order weighting average operators (OWA), see Ribeiro et al. (2000), Zmeskal (2002). (4) Financial modelling under soft input data and ill-structured decision-making circumstances, approach is based on assumption that input data (parameters, distribution functions) is possible introduce only vaguely. The approach might be seen as generalised sensitivity analysis, see Tanaka et al. (2000), Inuiguchi and Ramík (2000), Zmeskal (2001), Carlsson et.al. (2002), Sakawa et .al. (2003), Carlsson et.al. (2003), Yoshida (2003), Zmeskal (2004).

3. Fuzzy-stochastic portfolio approach

The approach to modelling uncertainty of portfolio mean variance methodology under soft conditions by fuzzy-stochastic methodology will be described. Risk is modelled by stochastic methodology on the portfolio basis and vagueness is modelled through the fuzzy numbers. It is shown, that methodology described could be considered to be generalised sensitivity analysis.

Portfolio model is of the crisp-stochastic category. It means that input data and parameters (goals, objectives, and constraints) are determined as crisp (real) numbers or unique distribution functions. Thus, it is supposed that decision-maker is able to exactly determine decision parameters and unique input data (distribution functions) are known. These assumptions are fulfilled in many situations and traditional portfolio optimisation method is useful methodology for prediction financial risk. However, because of complexity of financial systems, there are several situations that input data is not possible determine precisely but only softly. It means, decision-maker should not be able to state parameters (goals, constraints) crisply by crisp numbers or unique distribution function but only vaguely by fuzzy numbers or fuzzy probability distribution functions. Therefore, unique distribution functions are not at disposal. and it is a function of the weight attached to a probability judgment. Such conditions exist if financial system is non-stable or illiquid, e.g. in transition economies, emerging markets, after crisis, innovative companies, new technologies companies, long-term forecasted horizon etc.

Crisp-stochastic chance constrained M-V portfolio model

A special example of general chance constrained stochastic model is the M-V (mean-variance) model with normal distribution of random variables. The M-V version of Problem 1 is formulated as non-linear programming problem.

Problem 1 (mean-variance chance constrained problem with expected utility objective function)

$$E[U(\sum_j \bar{c}_j x_j)] = E(\bar{c}_j)x_j - r \cdot \mathrm{var}(\bar{g}) \rightarrow \max$$

$$\text{s.t.} \quad \sum_j^M E(\bar{a}_{ij})x_j \geq E(\bar{b}_i) + \Phi^{-1}(1-\gamma)\cdot\sigma(\bar{h}_i), \quad \text{for} \quad i = 1, M, \qquad \text{(P1)}$$

$$x_j \geq 0, \quad \text{for} \quad j = 1, N, \qquad \text{(P2)}$$

where $\mathrm{var}(\bar{g}) = \mathrm{var}(\sum_j^N \bar{c}_j x_j) = \sum_k^N \sum_l^N x_k \sigma_{kl} x_l$,

$$\sigma(\bar{h}_i) = \sigma(\bar{b}_i - \sum_j^N \bar{a}_{ij} x_j) = \sqrt{\sum_k^{N+1} \sum_l^{N+1} y_k \sigma_{kl} y_l}, \quad \vec{y} = [1, \vec{x}].$$

The objective function depicts maximum expected utility value in M-V formulation. By symbol r is expressed investors attitude to risk, vector x is optimised solution, $E(\bar{c}_j)$ and $\mathrm{var}(\bar{c}_j)$ are expected and variance values of coefficients.

By constraints (P1) is described, that inequality greater or equal with probability $1-\gamma$ should be fulfilled. Symbol $\Phi^{-1}(1-\gamma)$ means the inverse normal standardised distribution function of $1-\gamma$, σ_{kl} is covariance. Conditions (P2) are non-negative constraints.

Now, we suppose that random parameters may be introduced only vaguely by fuzzy-random variables. The objective function includes two aspects, stochastic (expected utility function) and vagueness (minimal fuzziness). Constraints include also stochastic aspect (probability level of constraint) and vagueness (fuzziness of binary relation). Therefore fuzzy-stochastic optimisation model is hybrid and fuzzy-stochastic modification of Problem 1 is following.

Problem 2 (generalised fuzzy-stochastic chance constrained M-V model)

$$\tilde{E}[U(\sum_j \tilde{\tilde{c}}_j x_j)] = \tilde{E}(\tilde{\tilde{c}}_j)x_j \tilde{-} r \cdot \mathrm{var}(\tilde{\tilde{g}}) \rightarrow \tilde{\max}$$

$$\text{s.t.} \quad \sum_j^M \tilde{E}(\tilde{\tilde{a}}_{ij})x_j \;\; \tilde{R}G \;\; \tilde{E}(\tilde{\tilde{b}}_i) \tilde{\mp} \Phi^{-1}(1-\gamma) \cdot \tilde{\sigma}(\tilde{\tilde{h}}_i), \quad for\ i = 1, M, \qquad \text{(P3)}$$

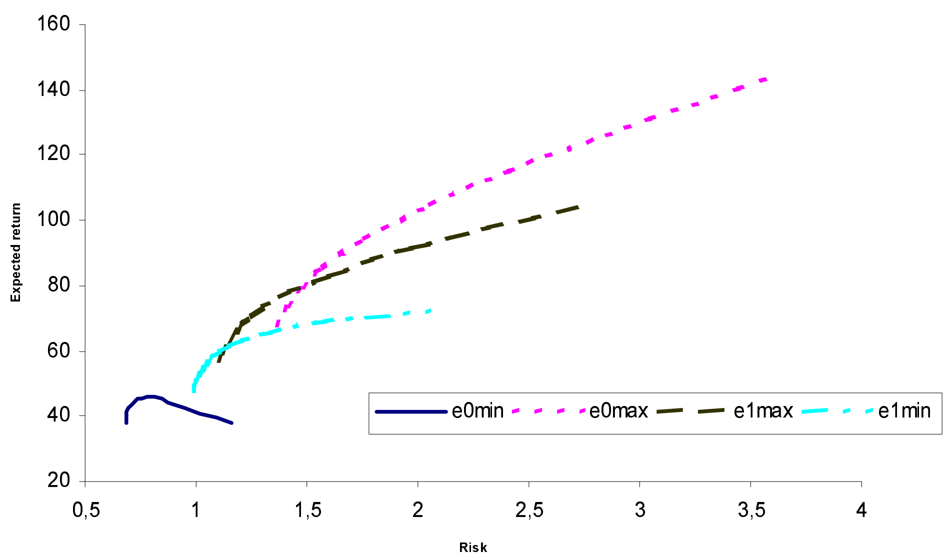
$$x_j \geq 0, \qquad \text{for } j = 1, N, \qquad \text{(P4)}$$

where $\tilde{\mathrm{var}}(\tilde{\tilde{g}}) = \tilde{\mathrm{var}}(\sum_j^N \tilde{\tilde{c}}_j x_j) = \sum_k^n \sum_l^n x_k \tilde{\sigma}_{kl} x_l,$

$$\tilde{\sigma}(\tilde{\tilde{h}}_i) = \tilde{\sigma}(\tilde{\tilde{b}}_i \tilde{-} \sum_j^N \tilde{\tilde{a}}_{ij} x_j) = \sqrt{\sum_k^{N+1} \sum_l^{N+1} y_k \tilde{\sigma}_{kl} y_l}, \quad \vec{y} = [1, \vec{x}].$$

The objective function depicts maximum expected utility value in M-V formulation and fuzziness. By symbol r is described investors attitude to risk; variable x_j is optimised result. Symbols $\tilde{E}(\tilde{\tilde{c}}_j)$, $E(\tilde{\tilde{a}}_{ij})$, $\tilde{E}(\tilde{\tilde{b}}_i)$ are fuzzy expected values, and $\tilde{\mathrm{var}}(\tilde{\tilde{g}})$, $\tilde{\sigma}(\tilde{\tilde{h}}_i)$ depict fuzzy variance and standard deviation, symbols $\tilde{+}, \tilde{-}, \tilde{\max}$ are for fuzzy additive, subtraction and maximisation operations, $\tilde{R}G$ .is fuzzy binary relation of greater or equal.

The problem should be solved under normal fuzzy set by decomposition principle with in α-level cuts. Graph shows efficient sets of the M-V model under soft conditions due to Problem 2 in α-level cuts.



4. Conclusion

A traditional conception of financial modelling is based on the idea the fuzzy approach is additionally inserted into a model. However, the opposite is true, a crisp-stochastic model being something like an inter-member.

A resulting resolution is simultaneously a sensitivity analysis of a decision-making model. From this point of view, the described fuzzy-stochastic modelling method is a common sensitivity analysis comprising elements of roughness. Thus a fuzzy model can be considered to be a generalized approach towards modelling in a economic and financial sphere.

It was shown that fuzzy-stochastic models reflect financial decision-making conditions and include aspects, randomness (stochastic) and vagueness (fuzziness). Therefore, the hybrid model might be considered as a generalised approach and advanced generalised sensitivity analysis.

Applications of the stochastic, fuzzy or fuzzy-stochastic models are not in contradiction. Each of these models is suitable for different decision-making conditions. Suitable applications depend on if input data of the model coincide with decision-making circumstances and input data preciousness.

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