

**Resolution of Puzzles on Dividend Policy,
Capital Structure and Corporate Control**

Kazem Falahati

Glasgow Caledonian University, Glasgow

G4 0BA, Scotland, UK

Tel: (+44) 141 331 3708, Fax: (+44) 141 331 3171

E-mail: kfa@gcal.ac.uk

ABSTRACT

For a company operating in the standard frictionless market with information symmetry and no moral hazard, the conditions for having optimum amounts of equity and debt are identified. The take over value of a company will exceed its flotation value significantly, when the company does not follow an optimum financial policy for its dividend payouts or capital structure.

Key words: Corporate control premium, dividend policy and capital structure

JEL Classification Codes: G 30; G32; G35

1. Introduction

This paper follows the author's paper on "Reviewing Standard Finance", presented at this conference. The latter is a prerequisite for understanding this paper. Section 2 deals with the dividend policy and capital structure of the firm, and Section 3 deals with corporate control premium in the frictionless market (in its standard sense). The Appendix develops an axiomatic formal model for the new paradigm in finance where the law of one price does not hold in this market.

2. Dividend and Capital Structure Policies

This part of the paper identifies the conditions for having optimum policies on dividend payouts and capital structure for a firm in a frictionless market (in its standard sense) with information symmetry and no moral hazard. The earlier paper proved that in the standard frictionless market the law of one price does not hold.

Following the break down of the law of one price in the standard frictionless markets, it is assumed that any company is operating in a competitive market where the price of each good at any date has a spread with an upper and lower bound. Competing arbitrageurs determine the *maximum* spread between these bounds at any date.

Competition amongst arbitrageurs ensures that the maximum spread for any good and at any date will be determined by the arbitrageur

with the *minimum* cost of capital in an instance of trading starting at that date. This is the cost of capital that is tied up in a single round of arbitrage in the good. Thus, at any date in any state of the world for the same good, there will be a range of prices within a *closed interval* representing such a maximum spread.

Subsections 2.1-2.3 describe further characteristics of this market.

2.1 PERMANENTLY CIRCULATING CAPITAL OF A FIRM

In exactly the same way that a farmer needs to sow the same kind of seeds that she reaps, a firm needs financial capital i.e. money as input to produce the same as output. However, as this is a “seed” that needs to be *continuously* ploughed into the business, it leads to the requirement for the firm to maintain a certain amount of capital invested *permanently* to continue its operations as a *solvent* concern without any interruption.

For the firm, at its inception, this capital represents the initial investment outlay for one input-output reproduction cycle. Thereafter, it can be augmented by the returns from trading, reinvested in the firm to expand the business. Clearly, if the business leads to losses, it will contract. Funds superfluous to the investment plans of the company go to shareholders as dividends.

Hereafter, the buying price of the entire investment for one input-

output reproduction cycle is called the *permanently circulating capital* (*PCC*) of the firm. The *PCC* could thus include cost of materials, labour, rent of land, hire of machinery, royalty for know-how and financing expenses e.g. cost of credit given, insurance, etc. all for a *single complete input-output cycle*. Clearly, the size of this capital can be variable over time, although it never disappears completely as long as the company is trading.

The *PCC* thus finances the *core operational investments* of the firm. Other investments of the firm which are not vital for its immediate reproduction cycle are called *peripheral* investments. Clearly, given that *peripheral* investments of the firm are not vital for its survival by definition, whoever controls the *PCC* of the company will control the core reinvestment programme of the firm, and hence will control the company to the extent that it is feasible.

The *PCC* of all traders provides the flow of *liquidity* which runs throughout the market, and is akin to the permanently circulating blood of a live body for an active economy.

2.2 DISTINCTION BETWEEN DEBT AND EQUITY

For the purposes of this paper, scarce private goods are to include, as well as ordinary goods, money and service contracts for all employable work. Thus, an individual may have no other scarce private good than her own labour power. Scarce private goods in any

instance can be consumed (used), invested, or held as capital tied-up in transit during an exchange. If held in the latter form, they will be part of the *permanently circulating capital (PCC)* of the economy.

Debt-holders of an individual require, by means of a contract with the individual, that their investment is repaid within a fixed time table. In contrast, there is no *contractual timing constraint* to repay *own* equity.

An individual's *wealth* at any *date* comprises scarce private goods that she either *already* owns or she *currently* holds in the form of capital tied-up in transit and thus will own prospectively, all of which are her *assets*. Her *wealth* will also include her *debts*: *i.e.* scarce private goods that she *owes* *i.e.* must give to others prospectively. The individual's *equity* will be the monetary value received from the sale of her assets less the monetary value paid to settle her *debts*.

Shareholders' investment in the company is the *equity* of the firm. Each share in the company carries a voting right which gives the shareholder a say over the control of the company. In the absence of any other claim to the ownership of the *PCC* of the firm, the collective united body of the shareholders has the right to control the company.

Debt-holders of a firm require, by means of a contract with the company representing the united body of its shareholders, that their investment is repaid within a fixed time table. In contrast, there is no *contractual timing constraint* to repay the shareholders' investments as

long as the firm honours its debts.

No market participant can have a negative equity, as the latter implies default. Thus, all market participants seek to minimise the possibility of default to the extent that this is within the scope of their control. They also seek to maximise the return on their equity within the same scope of control.

If a company does not honour its debt-holders' contracts in any way, the collective united body of debt-holders will have the right to seize control of the company, and gain access to all of its assets and use it to repay the debts of the company insofar as this is possible. Subsequently remaining surpluses will go to shareholders, and if the company is unlimited, shareholders will bear any remaining deficits to the extent that their equity endowments permit.

2.3 OPTIMUM FINANCIAL POLICIES

As explained in Proposition 2 of the earlier paper, "Reviewing Standard Finance", default risk can never be fully avoided due to events outside the control of the united body of shareholders. For, it is not possible to insure against *all* risks even in a perfect market. *In fact, a market for all risks cannot have a formal representation which is internally consistent and complete.* This follows *Gödel's incompleteness theorems*. The standard paradigm overlooks the huge implications of

these theorems in its construction of a perfect and complete market.

The least optimistic prospective market participant, who expects to exchange a good, is defined as the one who expects to obtain the least favourable price for it. That is to say she expects that she will sell the good at the lowest price *or* buy it at the highest cost within the range of prices within the maximum bid-ask spread available at any date.

The probabilities of the states of the world which affect the cash flows of the company are assumed to be known to all in this market. The forecast net mean cash flows of any financial decision are computed from the perspective of the least optimistic existing shareholder of the firm, and are based on these probabilities.

It is assumed that the least optimistic existing shareholder can determine her minimum expected return on a given project, hereafter this is referred to as her *required* mean return. It is also assumed that she can determine, from her own perspective, the mean return *offered* on that project yielding at least zero net present value.

An *optimum financial policy for a firm* is defined from the perspective of its existing least optimistic shareholder, who will have the highest cost of capital amongst all the shareholders of the firm. The policy thus must generate the greatest return expected by any of its shareholders *and* minimise the default risk of the firm. The latter means reducing the possibility, to the extent that it is within the scope of control of the

collective united body of shareholders, that the debt-holders of the firm take control of it.

Proposition 1: *For a company in the standard frictionless market, it is proved that its PCC (permanently circulating capital) must always be equity financed to reduce its default risk to its absolute minimum. Moreover, its optimum dividend policy must be such that, from the perspective of its least optimistic shareholder at any time, it will recover the selling price of its PCC.*

Proof: Before identifying an optimum dividend policy, let us first prove that in order to minimise the risk of default for a company or its shareholders it is only equity that can finance the PCC of the firm. Recall that the PCC represents the required funds for just one reproduction cycle of the company at any time.

It is already clear that whoever controls the PCC of a company will control its reinvestment programme, and hence will control its future to the extent that it is feasible and vital to its survival. Thus, if the united body of shareholders is to have the right to control the company at all times, it must then fund its PCC permanently.

The PCC of a firm is *permanently* tied-up in its reproduction cycle, and cannot be used for any other purpose, e.g. repayment of debt. In particular, it is not possible to have a perpetually revolving loan on acceptable terms at all times to finance the PCC of a company. For under uncertainty, there is no guarantee that any market participant

at any time can borrow any amount of money at an acceptable rate of interest. Thus, it is not possible for the debt-holders to fund the *PCC* of the firm at any time. This will reduce the default risk of the company to its absolute minimum.

The individual must thus draw from her endowment of equity to support the *PCC* of a firm at all times, which will in turn become the equity of the firm. This minimises the possibility of default for shareholders and reduces the likelihood loss of control of the company to any debt-holder. That is such control that is already within the grasp of the united body of the shareholders.

It follows that the firm will always have to retain its *equity funded PCC* even in the infinite horizon to ensure its solvency. However, from the perspective of its least optimistic shareholder, the *present value* of this *PCC* remaining in the infinite horizon must be infinitesimal to ensure an optimum dividend policy for the company.

Thus, the optimum dividend policy of the firm at any time requires that the current selling price of the assets, from the perspective of its existing least optimistic shareholder, be recovered from the present value of all of its prospective dividends, whilst maintaining the solvency of the company. If this were not so, it could then lead to the emergence of corporate control premium, and the possible loss of the control of the firm to a specialist investor.

The shareholders will thus fund the *PCC* of the firm insofar as this investment not only meets the least optimistic shareholder's expected total return requirement but also leads to an optimum dividend policy for her. If these conditions are met, then the shareholders' maximum contribution will determine the maximum scale of the *core* operations of the firm i.e. the optimum size of the *PCC*. The latter will thus be the *optimum equity investment (OEI)* for a single cycle of its operations.

Clearly, there will be a point for the least optimistic shareholder when her *required* mean return on further expansion of this investment project will exceed any mean return *offered* on this project. Otherwise, there will be no limit to the scale of investment projects that can increase shareholders' equity, contrary to the assumption of scarce resources. This constraint leads to the existence of a maximum scale of investment for any shareholder and a maximum scale of operations for a firm, which are determined concurrently.

These are the conditions required for the *OEI* of a firm; however, the existence of the *OEI* depends on the existence of an optimum policy for the firm. There can be many optimum dividend policies. On the other hand, such a policy may not exist. If so, the firm may well be taken over by an investor who is content with this investment and any of its indefinite retention of funds. Finally, if there is no such investor, this firm cannot exist within the capitalist system. (This provides a rationale for the existence of other forms of ownership of firms, e.g. co-operative, mutual, etc. where non-financial criteria my

override financial aims.)

The optimum use and amount of debt is addressed in Proposition 2. The *OEI* determined here remains valid under *any* financial policy. In particular, if the company raises debt, this policy will ensure that the firm will retain an optimum scale for its core operations at all times, and the control of the company will not pass to debt-holders.

Proposition 2: *For a company in a frictionless market with information symmetry and no moral hazard, and which has an optimum dividend policy, the existence of an optimum capital structure is proved.*

Proof: Let us first see how an optimum capital structure is obtained and later prove that it must exist. In the quantification of the cost of the *PCC*, we looked at the funding of the inputs of the company for just a single reproduction cycle at any time. However, it can be advantageous for a firm to secure its sources of input beyond a single reproduction cycle i.e. for the long term or permanently.

For example, it can be beneficial, for the long term, to buy rather than rent the land the firm currently occupies for its operations. This represents, in the terminology of this paper, a *peripheral* change to the *core* operational investments of the firm to secure long term inputs vis-à-vis investing merely in *PCC*, which is just for *one* reproduction cycle.

Subject to maintaining the *OEI* at its pre-determined optimum level at

all times to finance the *PCC*, the firm can raise *debt* for the *incremental* funds needed for these peripheral changes to its core operational investments. The financial impact of peripheral changes to core operational investments and changes to capital structure must be evaluated *together* from the perspective of the existing least optimistic *shareholder* to see if *as a whole* they offer added net present value to her.

For example consider the case of using debt to replace *rented* land by *owned* (by the firm) land. Firstly, this needs computing the net mean cash flows after interest expense on debt within a finite horizon in which the land is sold and the debt is repaid. Secondly, it requires discounting the net mean cash flows from the perspective of the existing least optimistic shareholder. Thirdly, if the net present value of this project is successful, implementation will begin.

Moreover, there will have to be continual reassessment to ensure the viability of this project in the light of changing circumstances. In particular, just before the end of the finite horizon of this project, a re-evaluation must take place to see if the life of this project can be extended with or without modifications.

The peripheral changes to core operational investments must give *priority* to those investments with the least default risks for the firm. This implies that for example, investment in land may precede research and development, assuming the latter is riskier i.e. more likely to lead to the possibility that debt-holders will control the firm.

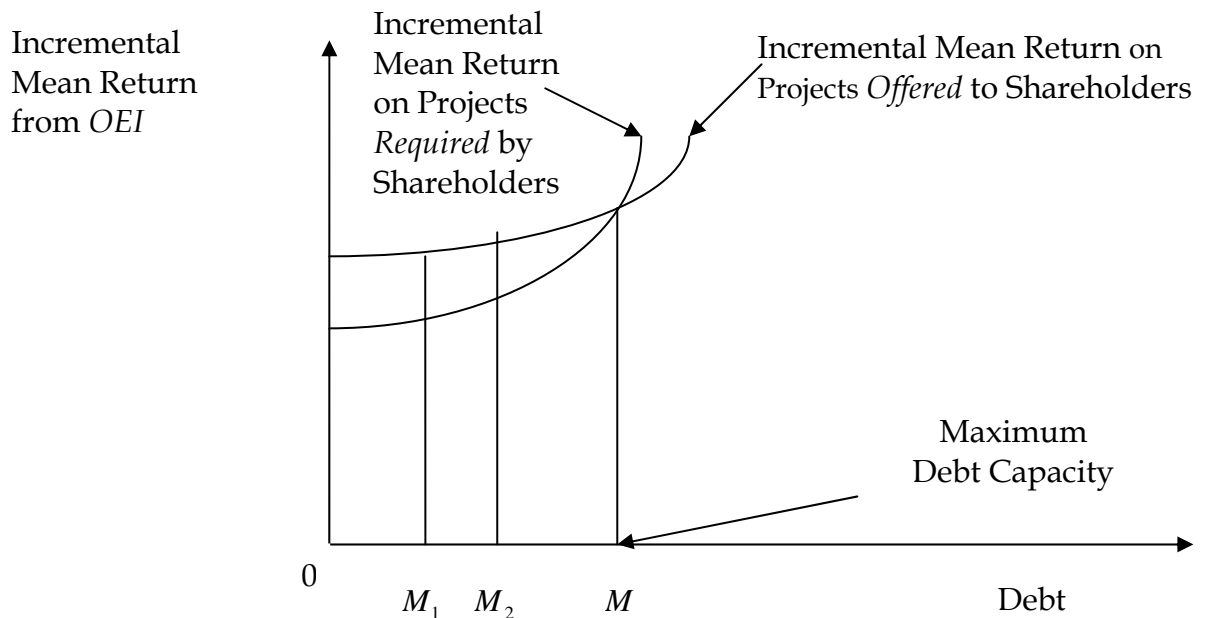
By accumulating riskier peripheral operational investments *and* increasing the level of debt for their incremental financing, the shareholders' *required* mean return will increase. A point will emerge when for any project the least optimistic shareholder's *required* mean return on their *equity* investment will be greater than the *offered* mean return on any investment. Otherwise there will be no limit to the increasing shareholders' equity. The maximum amount of debt is thus determined just before reaching this point.

Finally, it is worth noting that risk assessment is inherently subjective in this paradigm, hence it is unlikely that the optimum debt-equity structure of a firm will be the same for all investors (see Section 3).

The Appendix provides an alternative proof of Proposition 3 in a formal model, designed for the new paradigm, where the law of one price does not operate in the standard frictionless market.

Example: The following Exhibit illustrates the existence of this point of maximum debt for a more specific case. All computations are from the perspective of the least optimistic shareholder, this helps set the risk parameters appropriately. Thus, to reduce the possibility of debt-holders controlling the firm, the *OEI* is set at its upper endpoint and the maximum amount of debt *is* set in terms of its lower endpoint. The exercise is continually re-evaluated with changing circumstances.

Exhibit: Projects in Peripheral Changes to Core Operational Investments to Secure Inputs beyond a Single Reproduction Cycle, Financed by *Incremental Debt* and a *Constant Optimum Equity Investment (OEI)*



For the firm, M_1 and M_2 represent different levels of debt relating to different projects representing cumulative peripheral changes to its core operational investments. Point M is the maximum or optimum point, whilst the left hand side of M represents inefficiency, its right hand side carries risk of loss of control to debt-holders.

Finally, these evaluation processes must be continually re-examined from the perspective of the existing least optimistic shareholder of the firm at the time of evaluation to maintain optimality of the policies for all shareholders at all times. This continual readjustment may never reach its target accurately, as the target is *moving* in a dynamic economy. Thus the evolution of such optima over time will resemble meandering in a corridor rather than moving along a line of no width.

3. Corporate Control Premium

Propositions 1 and 2 suggest that if the dividend policy or capital structure of a firm is not optimum; for its shareholders, a significant difference between the take over and flotation value of the firm arises. Indeed, the company may well be operating inefficiently not only financially, but also its core operations may not be at its optimum size.

Thus from the perspective of an equity investor, either of the first two following relationships for a firm can arise, and the third holds by deduction:

$$\textit{Take over Value} = \textit{Value of All Net Assets}$$

$$\textit{Flotation Value} = \textit{Present Value of All Prospective Dividends}$$

$$\textit{Control Premium} = \textit{Take over Value Less Flotation Value.}$$

Leaving inefficiencies aside, in this paradigm, the optimum policies are decided from the perspective of the least optimistic shareholder of the company. Therefore, all other things being equal between two firms, differences in perception of the least optimistic shareholders about risk can lead to different valuation of projects and firms.

Moreover, there is no reason in this paradigm to assume that the least optimistic shareholder of one firm is the same as any other. Control (take over) premium can thus reflect differences in their perceptions.

4. Conclusion

The paper identifies the *permanently circulating capital (PCC)* of the firm which, as long as the firm is a going concern, is never distributed to shareholders. It points out that in order to minimise the default risk of the firm, this capital must always be equity financed. Its value can have any positive magnitude in the infinite horizon. However, the present value of this magnitude in monetary terms, for a firm with an optimum dividend policy, must be negligible.

The conditions for having the optimum amounts of equity and debt are identified for a firm. These will maximise shareholders' expected return and minimise the risk of default to the extent that this is within the scope of control of the united body of the shareholders. The procedures for making optimal financial decisions are presented.

This paper considers a frictionless market in the sense used in the standard paradigm. It explains that in the absence of optimal financial decisions, the present value of prospective dividends of a firm represents the flotation value of its equity in such a market, and that this can differ from the current economic value of the net assets of the firm, with the latter representing its take over value or break up value.

It also explains that control (take over) premium can also reflect differences in perceptions.

APPENDIX: A FORMAL MODEL FOR THE NEW PARADIGM

In a formal model at any date the prices that an asset can take or the rates of return it may offer will each be a *set* of numbers in a closed interval in this new paradigm. Let the signs \succ and \prec reflect the *inequations* of this model. Thus, when an asset trades at positive prices P in a closed interval we write $P \succ 0$, although each transaction will take place only at one of the prices indicated by P . Similarly if the prospective rates of return R on an asset are positive we write $R \succ 0$, where R is a closed interval, despite the fact that the realised rate of return will be a number which may or may not be in this interval. Nonetheless, in this model, economic parameters such as P and R follow the same axioms, which will be specified shortly.

If asset P trades in a closed interval with both endpoints below those of asset Q , we write $P \prec Q$, and we say P is cheaper than Q . Conversely, if P trades in a closed interval with both endpoints above those of Q , we write $P \succ Q$, and we say P is dearer than Q . However, if neither $P \succ Q$ nor $P \prec Q$ holds, then we write $P \langle \rangle Q$. Thus, when two assets P and Q trade within the same closed interval $[m, n]$ then we write $P \langle m, n \rangle Q$, if the interval exists but it is not specified we write $P \langle \rangle Q$. In each case P is said to be at the same *level* as Q or conversely i.e. Q is at the same *level* as P .

Similar relationships can be defined in terms of rates of return on

assets for the same period. For example, when the rates of return r_A on asset A are *higher, lower or at the same level* than the rates of return r_B on asset B , we can write $r_A \succ r_B$, $r_A \prec r_B$ and $r_A \langle r_B$ respectively.

Arithmetic operations i.e. additions $P + Q$ and subtractions $P - Q$; or multiplications $P \times R = PR$ and divisions $\frac{P}{R}$ are defined with respect to *every* pair of numbers that P and Q or P and R take. Thus, the result from each of these operations may well have a different closed interval than P , Q or R . In fact, the endpoints of the closed interval for the new economic parameter, representing the result of these operations, will be the maximum and minimum numbers obtained from the set of results from each such operation.

It should be borne in mind that when a transaction such as a purchase is indicated by an addition of an asset and the subtraction of another asset, it may take at least an instance for this transaction to become valid. In such cases, the measurements at any date must take account of the capital tied up in transit, if relevant. However, an addition or subtraction of assets may not necessarily indicate a purchase or a sale, e.g. it can arise from a re-assignment from one department to another department within the same firm.

The following axioms are assumed to hold when measurements for each positive parameter are made at the same date: $I. 0 + P \langle P$,

II. $P + Q \langle \rangle Q + P$, III. $P + (Q + S) \langle \rangle (P + Q) + S$, IV. $P \times R \langle \rangle R \times P$, V.

$$\frac{P \times R}{P} \langle \rangle R,$$

$$\text{VI. } (P + Q) \times R \langle \rangle P \times R + Q \times R,$$

VII. If $P \langle \rangle Q$ then $P - S \langle \rangle Q - S$, VIII. If $P \langle \rangle Q$, then $P \times R \langle \rangle Q \times R$, IX.

If $U \langle \rangle V$ and $V \langle \rangle W$ then $U \langle \rangle W$, X. The sign of equality i.e. = retains

its normal arithmetic meaning. Thus $Pk = kP$ and $\frac{Pk}{P} = k$ when k

is an ordinary number e.g. a non-zero scale factor.

Let us look at an example to see how this model works. For a firm, there is at least an instance time lag for the receipt of the sale proceeds of assets to pay off the debts, during when prices can change for both assets and debts. Moreover, at any date in this new paradigm, an asset may well be tied up in transit, and there exists a multiplicity of prices for both assets and debts. Thus, the economic “*balance sheet*” *relationship* in the new paradigm between assets on the one hand, and on the other hand debt and equity cannot be an *identity*, in contrast to the standard view.

Indeed, there can be a spread between these two values indicated by a closed interval. Proposition 1 has shown that the length of this interval depends on the dividend policy of the firm. An optimum dividend policy can minimise this length, and may even make it negligible.

The “*balance sheet*” *relationship* for an individual or a firm in the new

paradigm is thus true within a closed interval on either side of the “balance sheet” i.e. $A \langle D + E$. This is where the prices of assets, debt and equity are indicated by A , D and E . Thus, if $B = D + E$ and $A \langle D + E$, then $A \langle B$ and $r_A \langle r_B$. Also, let us see how the inequations work by noting that based on the earlier axioms: If $P \langle Q$ and $Q \langle R$ then $P \langle R$; and $P + R \langle Q + R$; and when $R \rangle 0$, then $PR \langle QR$, and if $r_A \rangle r_D$, then $r_A - r_D \rangle 0$. A better idea as to how this model can be helpful is obtained from the following.

Proposition: *For any firm, with the expected rates of return generated by its assets higher than those required on its debts, and an optimum dividend policy already in force, there exists an optimum capital structure.*

Proof: Under the new paradigm, the “balance sheet” relationship for a firm with an optimum dividend policy is $A \langle D + E$ where A , D and E represent the prices of the assets, debt and equity of the firm respectively at the same date. If this holds, the offered mean rate of return on the assets of the firm r_A will be at the same level as the required mean rate of return on its portfolio of debt and equity.

The latter is $\{[(\frac{E}{D+E}) \times r_E] + [(\frac{D}{D+E}) \times r_D]\}$ where the required mean rates of return on: the debt is r_D and on the equity is r_E , and by assumption $r_A \rangle r_D$. Thus $r_A \langle \{[(\frac{E}{D+E}) \times r_E] + [(\frac{D}{D+E}) \times r_D]\}$, by

multiplying both sides by $D + E$, based on the axioms of this model, we obtain: $(D + E) \times r_A \langle E \times r_E + D \times r_D$, and now we can divide both

sides of the latter by E and write: $(\frac{D}{E} + 1)r_A \langle r_E + \frac{D}{E}r_D$, and by

subtracting $\frac{D}{E}r_D$ from both sides we finally derive:

$$r_E \langle r_A + \frac{D}{E}(r_A - r_D).$$

Given that the expected rates of return generated by its assets are higher than those required on its debts, then $D \rangle 0$. Assuming maximum debt has not yet been obtained, let h be the higher endpoint in the closed interval of $\frac{D}{E}$. The following proves that the

maximum amount of debt that the firm can raise is obtained when a rise of Δh in the debt-equity ratio generates an increase of Δr_D in the rate of interest payable on the loan such that $\Delta r_D \langle (r_A - r_D) \times \frac{\Delta h}{h + \Delta h}$.

The reason is any further increases in Δr_D such that

$$\Delta r_D \rangle (r_A - r_D) \times \frac{\Delta h}{h + \Delta h}$$

will require a lowering of shareholders expectations for the rate of return on their equity stake. For, as h and

$$\Delta h$$
 are scale factors, and $r_A \rangle r_D$, it follows from $\Delta r_D \rangle (r_A - r_D) \times \frac{\Delta h}{h + \Delta h}$

that $h \times \Delta r_D + \Delta h \times \Delta r_D \rangle \Delta h(r_A - r_D)$ or $0 \rangle \Delta h \times (r_A - r_D - \Delta r_D) - h \times \Delta r_D$ if

$r_A + h(r_A - r_D)$ is added to both sides of the latter, one obtains

$$r_A + h(r_A - r_D) > r_A + (h + \Delta h)(r_A - r_D - \Delta r_D).$$

It can now be seen that the left hand-side of this last inequation is at the same level as the previous *required* mean return on equity at the highest point of the debt-equity ratio and its right hand-side is at the same level as the new *required* mean return on equity, following the increase of Δh in the debt-equity ratio. Hence, any further increase in debt will only be possible if shareholders lower their expectations of their *required* mean return. However, that will be unjustifiable, given the increase in the financial risk of the company arising from an increased debt-equity ratio.