

Reviewing Standard Finance

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ABSTRACT

This paper identifies hidden contradictions in the standard paradigm which is characterised by the law of one price in frictionless markets. It proves that existing literature cannot offer internally consistent methods of valuation of companies, and leads to the break down of theorems of Modigliani and Miller and asset pricing. More generally it rigorously disproves the free arbitrage proposition.

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JEL Classification codes: *G12; G32; G35*

The “law” of one price in frictionless markets takes its origin from Aristotelian reciprocity and “justice in exchange”. It has dominated economic theory for centuries due to its normative appeal; hence shifting opinion on this point is a Herculean task. Nonetheless, in the belief that *unbiased* scientists accept superior reason even if it gives rise to a conflict of interest for them, this paper proves rigorously that this law is invalid when money has a positive time value or risk taking requires reward. This paper proves that the standard paradigm in finance, characterised by the law of one price, suffers from deep-seated inconsistencies. It demonstrates this point initially with reference to the Modigliani and Miller theorems, which rely on this law, to indicate the motivation for this paper. It later provides more general proofs to this effect, touching foundational issues in finance and general equilibrium.

This paper is concerned with the internal consistency of the standard paradigm and not its external consistency i.e. its realism. In the standard frictionless market considered in this paper, transactions of goods for money incur no costs for labour, storage, transport, communication, and there are no information asymmetries, agency costs, moral hazards or taxes.

Proposition 1: *Standard finance theory under the Modigliani & Miller dividend irrelevance argument cannot offer internally consistent methods of valuation of companies.*

Proof: Consider two all-equity companies in the standard paradigm in a perfect and certain market and given full information, with one company

having a greater amount of invested assets in monetary terms than the other. The economic balance sheet for each company will show:

$$\text{Assets} = \text{Equity}$$

Therefore, the equity value of one company is greater than the other.

Now, suppose both companies choose to have exactly the same dividend policy in terms of amount and timing for ever, which they can both afford to pay. It follows that the present value of the prospective dividends of each company will also be the same, thus both companies will have the same equity value, hence a contradiction.

It is important to note that in this perfect and certain market, these companies generate the same rate of return on assets. Nonetheless, the larger firm always pays a lower proportion of its return as dividends, and thus accumulates greater capital, and investors are fully aware of these facts. However, following the view that dividend policy is irrelevant to company valuation (Miller & Modigliani, 1961), by management choice, the dividend streams of both firms are made the same.

Clearly, the dividend valuation model cannot be wrong. However, it can be argued that if the larger company had chosen an optimum dividend policy, such a disparity between the asset and equity values would not arise. In fact, given the absence of any agency costs, shareholders will force an optimum dividend policy such that this disparity will disappear, or liquidate the company. However, this contradicts the Miller & Modigliani (1961) view that dividend policy is irrelevant to the valuation of companies.

It is worth noting that the Miller & Modigliani (1961) proof of the dividend policy irrelevance theorem rests on the upholding of the “law” of one price for the shares of the company. However, the assumption of dividend policy irrelevance generates one price for the stock market flotation value of the company represented by the capitalised value of its dividend stream, and another price for its takeover value represented by the liquidation price (or break up value) of its assets, hence a contradiction.

This is symptomatic of a paradigm which is inherently self-contradictory. To see the contradiction implied by Miller & Modigliani (1961) more clearly, consider two companies employing constant returns-to-scale technologies in this perfect and certain market. Let us assume the initial equity fund invested in the small company is $F > 0$ and the amount invested in the large company is $2F$ at the beginning of the first year, and the annual rate of return on these funds is $r > 0$. The dividend payout for each company at the end of year i is $D_i = F(r - g)(1 + g)^{i-1}$ where $i = 1, 2, 3, \dots$ and $r > g > 0$ by assumption.

It follows from the standard dividend valuation model that the present value of all the dividends of each firm tends to F , whilst the current value of the assets of the small and large companies are F and $2F$ respectively.

It is worth noting that at the end of year i , the equity fund of the small firm after dividend payouts is $F(1 + g)^i$ and that of the large firm is $F(1 + g)^i + F(1 + r)^i$. Thus, the *present* value of the retained equity fund of

the small firm, remaining inside it in the infinite horizon, is negligible whilst that of the large firm tends to F .

Proposition 5 illustrates how this problem arises in the context of general equilibrium theory, when the assets are reproducible capital goods, and notes the break down of the law of one price in that context.

Proposition 2: *In risky perfect markets where no one can borrow an infinite quantity of money or any other asset the value of the firm cannot be invariably independent of its capital structure.*

Proof: Let us assume that the expected rate of return for the firm on its assets is r_A , debt is r_D and equity is r_E , and A , D and E represent the prices of the assets, debt and equity of the firm respectively. Modigliani & Miller (1958) hold that the value of the (assets of) firm is independent of its capital structure (their Proposition I): $A = D + E$ irrespective of whatever positive value the debt-equity ratio $h = \frac{D}{E}$ takes. Subsequently they derive (their Proposition II): $r_E = r_A + h(r_A - r_D)$. This is where r_A is invariable to changes in the mix of debt and equity, and r_E is higher than r_D , reflecting shareholders' compensation for the greater risk borne, compared to debt-holders. They consider that leverage represents financial risk.

Following the work of Stiglitz (1969) and Rubinstein (1973), it is generally believed that in a "complete" and perfect market with information symmetry and no moral hazard this model takes account of all risks

consistently. In particular, it predicts that as the debt-equity ratio $h = \frac{D}{E}$, reflecting the financial risk of the firm, rises then, the expected rate of return on equity r_E will also increase. However, this is not always true in a market where no one can borrow an infinite quantity of money or any other asset (see Proposition 3), as explained in the following.

Consider when a rise of Δh in its debt-equity ratio of the company generates an increase of Δr_D in the rate of interest payable on its loan such that $\Delta r_D > (r_A - r_D) \frac{\Delta h}{h + \Delta h}$. This rise in the rate of interest for the company can be justified by the increase in its projected financial risk. Under these conditions, Modigliani & Miller's model predicts that the expected rate of return on equity will fall, despite the fact that the financial risk of the company will increase.

Here is the reason why this model predicts the fall in the expected rate of return on equity: It follows from this inequality that $\Delta h \times (r_A - r_D - \Delta r_D) - h \times \Delta r_D < 0$, if $r_A + h(r_A - r_D)$ is added to both sides of it, one obtains $r_A + (h + \Delta h)(r_A - r_D - \Delta r_D) < r_A + h(r_A - r_D)$. It can be noted that the right hand-side of this last inequality represents the expected return on equity when the debt-equity ratio is h ; and its left hand-side represents the expected return on equity, following the increase of Δh in this ratio.

For example, when $r_A = 9\%$, $r_D = 5\%$ and $h = 40\%$, this problem arises if

for $\Delta h = 10\%$ one obtains $\Delta r_D > 0.8\%$. That is to say: if an increase in the debt-equity ratio from 40% to 50% leads to a rise in the rate of interest on the debt from 5% to over 5.8%, then the expected rate of return on equity will *fall* from its previous level at 10.6%.

Modigliani & Miller propositions cannot justify why shareholders should be satisfied with a prospective rise in the financial risk of the firm *and* a fall in their expected return concurrently. In order to maintain consistency with the treatment of financial risk in this model, the expected rate of return on equity must rise in this case. However, Modigliani & Miller (1958, p.273-274) deny this. They concede that:

“Economic theory and market experience both suggest that the yields demanded by lenders tend to increase with the debt-equity ratio...”

When companies increase their debt-equity ratios they suggest:

“...while the average cost of funds will tend to increase as debt rises, the average cost of funds from *all* sources will still be independent of leverage...” and

“...the increased cost of borrowed funds as leverage increases will tend to be offset by a corresponding reduction in the yield of common stock.”

This represents an inconsistent treatment of the expected return and risk for shareholders. However, remarkably they note that this ruins their Proposition II, whilst maintaining their Proposition I:

“Although Proposition I remains unaffected as long as the yield curve is the same for all borrowers, the relationship between common stock

yield and leverage will no longer be the strictly linear one given by the original Proposition II.”

This is impossible, as under the law of one price Proposition I: $A = D + E$ immediately leads to Proposition II, for from Proposition I: $(1 + r_A)A = (1 + r_E)E + (1 + r_D)D$, which simplifies to: $r_E = r_A + h(r_A - r_D)$.

It is to be noted that the analysis of this paper does not depend on the existence or a non-existence of a “complete” market. In fact, a market for *all* risks cannot have a formal representation which is internally consistent *and* complete. This follows Gödel’s (1992) well-known incompleteness theorems.

(In formal logic, Kurt Gödel (1906-1978) proves that there will always be logical problems of undecidability or incompleteness in a non-trivial axiomatic framework. Thus, any internally consistent formal model for predicting future states of nature will be incomplete or will have “states” the possibility of the existence of which are undecidable. That is to say the assumption of existence or non-existence of that state leads to contradiction.)

Here, bankruptcy risk can never be fully avoided, as it is not possible to insure against *all* risks even in a risky perfect market, as the possibility that *all* insurers will default cannot be ruled out in a formal model. The standard paradigm in economics overlooks the huge implications of

Gödel's theorems in its construction of a perfect and complete market.

Proposition 3: *Unrestricted borrowing of risk free or risky assets leads to the generation of a free lunch and contradicts the assumption of scarcity of resources. Models which rely on this assumption are internally inconsistent.*

Proof: Many models in finance rely on unrestricted (and thus unlimited) borrowing of risk free or risky assets. For example, Ross (1977) notes that the capital asset pricing model *requires* unrestricted borrowing of risk free or risky assets. Similar observations can be made on the standard option pricing models (e.g. Black & Scholes, 1973).

The availability of a "free lunch", given unlimited borrowing in risky frictionless markets, is admitted in the standard paradigm, e.g. in (Harrison and Kreps, 1979), (Harrison and Pliska, 1981) and (Heath and Jarrow, 1987). However, it is argued that constraints such as margin calls (Heath and Jarrow, 1987) or non-negative wealth (Dybvig, 1988) eliminate a free lunch, even when unlimited borrowing is permitted. This is not so, for the assumption of unlimited borrowing makes these "constraints" irrelevant, as it guarantees a free lunch, and hence such conditions can be easily met.

In particular, it must be recognised that it is the ability to borrow limitlessly that makes it possible to engage in the well-documented (Harrison and Kreps, 1979) case of the "doubling strategy". This strategy in raising the stake, to an *unlimited* amount if necessary, in a sequential game of flipping a fair coin and stopping on the first win, guarantees a free lunch. Moreover,

this free lunch can be reproduced endlessly to obtain an infinite amount of money in such a frictionless market of the standard paradigm. That is to say a “money pump” can be produced with no risk and no net investment, thus violating the fundamental assumption of scarce resources.

Formal models can hide contradictions. For instance, under the standard assumption of unrestricted borrowing, a borrower can always repay a loan, i.e. both its principal and interest, by re-borrowing, making the initial loan a free lunch. To see this, let us assume the initial loan is l_0 at date 0 and the total amount of its principal and interest are l_t at date t , and the amount re-borrowed at date t is also l_t . The borrowing level can reach infinity in any period (finite or not) which admits unlimited borrowing. Thus, the cash flow stream of the borrower in respect of her borrowing at dates $t = 0, 1, 2, 3, \dots$ will be:

Date	0	1	2	3
Cash flow	l_0	$l_1 - l_1$	$l_2 - l_2$	$l_3 - l_3$
OR:					
Cash flow	l_0	0	0	0

It is clear that this process of re-borrowing can continue unceasingly under the assumption of unrestricted borrowing, thus the borrower can end up with a sure free lunch of size l_0 . In fact, given scarcity of resources, unlimited borrowing is impossible, and models based on this assumption are internally inconsistent.

Proposition 4: *If money has a positive time value or there exists the possibility of loss of exchange value for goods over time, then profit-maximising arbitrageurs cannot force the law of one price in the standard frictionless market. Moreover, it is possible for the selling price of a good to exceed its buying price at the same date non-negligibly without generating a free lunch in a risk free or risky market.*

Proof: Existing literature assumes that the buying *and* the selling price of a scarce private good at any date must be the same in a frictionless market for the same trader; otherwise it gives rise to an arbitrage opportunity, which generates a “money pump”, i.e. an infinite amount of money. Dybvig and Ross (1987) define an arbitrage opportunity as follows:

“An arbitrage opportunity is an investment strategy that guarantees a positive payoff in some contingency with no possibility of a negative payoff and no net investment. By assumption, it is possible to run the arbitrage possibility at arbitrary scale, in other words, an arbitrage opportunity represents a money pump.”

The following proof demonstrates that arbitrage in standard frictionless markets can minimize but cannot eliminate the spread between the selling price of a private good and its buying price at the same date. This is when the time value of money is positive or prices are volatile. Moreover, it identifies the conditions in the standard frictionless markets where this spread *must* be significant i.e. *non-infinitesimal*, contrary to existing beliefs. (Here, it is appropriate to note that in mathematics, the infinitesimal is not admitted in standard analysis. In non-standard analysis (Robinson, 1996), which is drawn on in this proof, the infinitesimal is the inverse of the

infinite and can be zero or infinitely close to it.)

To do so we look very *closely* at the movements of goods and money over time in an exchange to differentiate between valid and invalid transactions used to justify free arbitrage:

It is impossible for the same individual (say, I_1) to buy from one party and sell to another individual (say, I_2) the same specific *item* of a given *private* good *concurrently*, (assuming each *item* is uniquely identifiable e.g. by a serial number). Otherwise *both* I_1 *and* I_2 *would be buying the same item of this good concurrently*, which is logically *impossible* by the definition of a private good. Therefore, this would be an *invalid* trade.

Moreover, a profit maximiser who owns a private good which offers a profitable arbitrage opportunity will take advantage of this opportunity herself rather than lend this good freely to others to exploit this opportunity by *short selling* it. Thus, any such lending is *invalid*, and such invalid transactions cannot be used to justify free arbitrage.

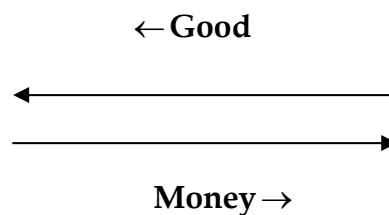
Let us now make a distinction between a *date* with no length in time and an *instance* which represents a positive infinitesimal quantity of time. For the same individual, the ownership of an object can start at a date *or* cease at a date, taking no time. However, one cannot start *and* cease the ownership of the same object, *in that order at the same date*, as that is self-contradictory.

Thus for the claim to the ownership of an object to be valid, ownership must take at least a *positive infinitesimal* quantity of time i.e. an *instance*, from the first date that such a claim may refer to. (Existing literature overlooks the significance of this point for continuous-time trading and risky markets. Its critical importance will become obvious later in the course of this proof).

In contrast, the loss of ownership of an object can be assumed to start *and* cease at the same date without self-contradiction. In fact, to make the contrary assumption (i.e. that the loss of ownership of an object takes a positive amount of time) leads to the view that the individual may still own the object in some way after the date it was lost, which is self-contradictory. Therefore, in conformity with the existing literature in this paper the loss of ownership of an object is assumed to occur at a *date*.

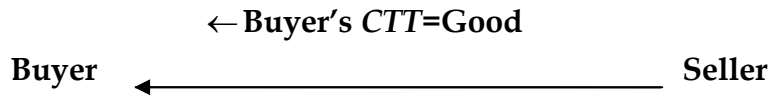
Hence, an exchange of ownership of objects must take at least an instance for the new ownerships to become valid. Exhibit 1 shows the length of the instance of exchange, infinitely magnified for convenient observation along two parallel lines (the long arrows with bold heads). They indicate the opposite and concurrent movements of the good and the money.

Exhibit 1: Instance of Exchange



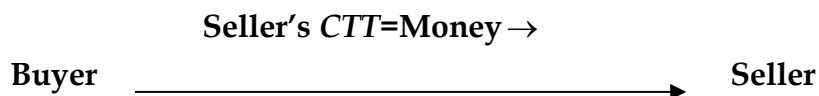
In a good-for-cash “spot” transaction, it takes at least an instance for the buyer of the good to own it validly, although the seller loses this good at the start of this instance of exchange. Thus, this good is *capital tied-up in transit (CTT) for the buyer* during the instance of exchange, when, she cannot use it in any other way, although she has a claim to its ownership.

Exhibit 2: Buyer’s Capital Tied up in Transit in the Instance of Exchange



It also takes at least an instance for the seller to own the cash representing the selling price of the good, although the buyer ceases to be the owner of this cash at the start of the instance of exchange. Hence, this cash is *capital tied-up in transit (CTT) for the seller* during the instance of exchange, when, she cannot use it in any other way, although she has a claim to its ownership. (It is worth noting any other form of contract other than the “spot” transaction requires the existence of the CTT on its delivery.)

Exhibit 3: Seller’s Capital Tied-up in Transit in the Instance of Exchange



Let us now define *valid arbitrage transactions*:

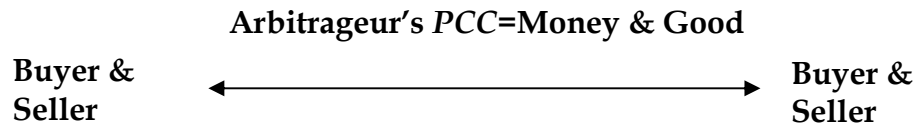
For arbitrage transactions to be *valid* in respect of a scarce private good with two prices at the same date, the arbitrageur must buy the cheap item and sell the dear item of that good in the same quantity *concurrently*.

It follows that an arbitrageur, who buys and sells different items of a good in the same quantity concurrently, will have her capital tied-up in transit in every instance of exchange in the form of money and good.

There will *always* be some traders who will engage in *continuous-time* arbitrage, for if they do not, the existing literature offers no guarantee for the law of one price to hold. For a trader who unceasingly engages in arbitrage during period P , her *CTT* will be capital tied-up *permanently* in transit. This capital will hereafter be referred to as *permanently circulating capital (PCC)*. The *PCC* of an active market (for economics) is analogous to the *permanently circulating blood* of a live body (for biology), as it provides *liquidity* to the market. It also generates a hidden friction in a market which the standard paradigm regards frictionless. (Nonetheless, this paper will continue to use the language of the standard paradigm regarding friction.)

Exhibit 4 reflects an instance of exchange for the continuous-time arbitrageur, acting as a buyer and seller concurrently. Let us assume her *PCC* will be $2K$ in terms of her buying prices at the start of period P when this process takes place, and that her required rate of return on capital is R during P . Given that the exchange value of this good in money terms is significant by assumption, her *PCC* of $2K$ will also be a significant amount.

Exhibit 4: Arbitrageur's Permanently Circulating Capital



As pointed out earlier, there will always be some traders who will engage in continuous-time arbitrage, for if they do not, the existing literature offers no guarantee for the law of one price to hold. However, continuous-time arbitrage leads to a significant PCC with a significant cost to the arbitrageur i.e. $2KR$ at the end of period P . The reason is P can easily be a period of such length that the trader's required rate of return R on her capital will be significant. The existence of a significant PCC during arbitrage makes it impossible for the arbitrageur to operate with "no net investment" or at a negligible cost during P . To offset this cost, the selling price must exceed the buying price at the same date for the arbitrageur, hence a contradiction.

The size of this excess depends on the quantum that R takes in an instance. The following studies this point firstly in a world without and later with risk. Let us assume that period P is divided into n consecutive instances in each of which trade takes place consecutively, and the discrete rate of interest in any of these instances is a positive infinitesimal equal to $\frac{r}{n}$.

This is in infinitesimal calculus, alternatively known as non-standard analysis (Robinson, 1996) where r is positive, significant and finite and n is infinite. Then the discrete rate of interest R for the duration of the full length of P will be a *significant* positive number for:

$$R = \left(1 + \frac{r}{n}\right)^n - 1 = e^r - 1, \text{ where } e = 2.718\dots$$

This is true even when P is an instance. Thus, where a single round of arbitrage transactions takes the full length of instance P , the cost of capital for the arbitrageur will be significant i.e. $2KR$ at the end of P . Hence, it is possible to have a significant spread of size $2KR(1+R)^{-1}$ between the buying and selling price of a good at the same date, which will not generate any free lunch.

In risky markets, it is easy to see that surprising news can change the share price of a firm *significantly* in an instance. Given that such a possibility always exists, it is fair to assume that arbitrageurs will require significant compensation for any instantaneous adverse price movements. This will be reflected in a significant spread between their buying and selling prices.

The following explains the reason for this. The buyer of a good must own it for at least an instance for her purchase to be valid. Given the possibility of a significant instantaneous price fall, she thus carries the possibility of a significant monetary loss if she sells it at the end of this instance. It follows that an arbitrageur in this good must *anticipate* the possibility of such a loss, and thus requires significant compensation for accepting this risk. She therefore must seek to cover this risk by a significant spread between her buying and her (higher) selling price at the same date. However, existing literature does not pick up this point, as it does not admit the need for the existence of an instance-long minimum ownership period for the purchaser.

This leads to an implicit lack of recognition of the *arbitrageur's trading risk*. This is true even if all prices of the same good at the same date are correlated. To see this, consider a market for a good in which there are different prices quoted for it at any date, and the random changes in prices for the same good from one date to next are *uncorrelated*.

Let us assume that the consecutive dates $t_0, t_1, t_2, t_3, \dots, t_N$ are each an instance (of equal length) apart. Consider a trader in this market who buys in one instance $t_{j+1} - t_j$ and sells in the next instance $t_{j+2} - t_{j+1}$ the same quantity of this good for immediate cash *continually* in two sequences as follows. Each sequence comprises a series of consecutive buying and selling.

The first sequence for which $j = 0, 1, 2, 3, \dots$ starts from t_0 and the second sequence for which $j = 1, 2, 3, \dots$ starts from t_1 . This leads to continuous-time arbitrage in this good from t_1 onwards, consistent with the conditions for valid arbitrage. The latter requires that for arbitrage transactions to be *valid* in respect of a scarce private good, the arbitrageur must buy the cheap item and sell the dear item of this good in the same quantity *concurrently*.

If we look at these dates along a timeline, where any infinitesimal length of time is infinitely magnified for convenient observation, the trader's cash flow along this line will show:

Exhibit 5: Arbitrageur's Cash Flow

Date	t_0	t_1	t_2	t_3	...	t_N
Out	$-B_0$	$-B_1$	$-B_2$	$-B_3$...	$-B_N$
In	0	0	S_1	S_2	...	S_{N-1}
Net	$-B_0$	$-B_1$	$S_1 - B_2$	$S_2 - B_3$...	$S_{N-1} - B_N$

Here, S_N and B_N represent respectively the selling and buying prices of the same good at date t_N . The arbitrageur loses the ownership of the cash paid for her purchases immediately, whilst it takes her an instance to own her sale proceeds. This leads to a net cash flow of $S_{N-1} - B_N$ at t_N . It is now easy to see that S_{N-1} and B_N are *uncorrelated*; and that the net cash flow of the arbitrageur at each consecutive date is not risk free.

Proposition 5: *General equilibrium models, by presuming the identity of the buying and selling price for the same scarce private good at the same date for the same trader in frictionless markets, generate contradictions.*

Proof: Excess demand, net supply and profit are defined in neo-classical economics assuming that the buying *and* selling price of the same good is equal for every trader (Arrow and Hahn, 1971) at the same date. This includes a trader who buys and sells the same good, e.g. a farmer who *reproduces* the same seeds that she grows, and can buy and sell her input and output of the same good infinitely many times, akin to an arbitrageur.

The following general equilibrium models each display a contradiction, as it is assumed in these models that the law of one price holds in terms of the present value of the same good at all times.

When considering *reproduced capital goods*, which producers can have both as input *and* output at the same price in terms of present value infinitely many times, Walras discovered that his general equilibrium model is over-determined, as Eatwell, Milgate and Newman (1990, p. xii) point out, and Eatwell (1990, p.252) explain. The reason is there is no guarantee that the cost of production of reproducible capital goods will necessarily equal the capitalised value of the rentals paid for their services, as determined by market clearing. (cf. Proposition 1).

Moreover, Eatwell, Milgate and Newman (1990, p. xiii) note that under certainty in a wide range of “intertemporal” models of equilibria, including the Arrow-Debreu:

“The rate of return on non-reproduced capital goods is *lower* than the rate of return on reproduced capital goods.” (Emphasis added).

However, the same models leave open the opportunity for free arbitrage, which requires a uniform rate of return for all risk-free investments in this paradigm, hence their fundamental contradiction.

A contradiction can also arise in respect of *reproducible consumer goods* as in the following case: Under the law of one price, Samuelson’s (1958) overlapping-generations model of general equilibrium leads to a significant

free lunch for the first generation. This is whilst other generations which *give and receive the same scarce goods at the same price* in present value terms make zero economic profit under the law of one price. Clearly, this free lunch can be reproduced endlessly to lead to a money pump, thus violating the most fundamental tenet of economic theory i.e. scarcity of resources.

Reason behind contradictions: Existing literature confuses the concept of a *date* i.e. a point of no length in time as against an *instance* i.e. a positive infinitesimal quantity of time. It thus assumes that the ownership of a good need not take even an instance to be valid. Hence, any change of ownership of goods arising from the action of trade, and in particular, an infinite number of consecutive trades are *invariably* presumed to take no time. This leads to overlooking the existence of, and the cost of, the *capital tied up in transit* during exchange.

This capital represents goods and monies the ownership of which are being exchanged and are impossible to consume, use or invest elsewhere in the instance of exchange. Continuous-time-traders will *always* have part of their capital tied up in transit in an exchange, representing their *permanently circulating capital*. *This overlooked capital flow is as indispensable to the functioning of a market as blood circulation is to a live body, and it provides liquidity to the market.*

If the goods being traded have significant money prices, this capital will also be significant in money terms. Given a positive time value for money, the continuous-time trader will suffer a significant cost due to her *capital tied up permanently in transit over a significant period*. Overlooking this cost to this trader is equivalent to assuming that these goods have no significant price, or money has no time value.

For the law of one price to hold permanently, existing literature implicitly requires traders to engage in *continuous* arbitrage interminably, for if arbitrageurs stop, it considers that there will be no guarantee for the law of one price to hold. However, permanent arbitrage leads to a significant amount of capital being tied-up permanently in transit which will have a significant cost for arbitrageurs. The only way arbitrageurs can recover this cost is through a spread between their buying and selling prices.

This spread can be significant particularly in a risky market. Thus, in a free market, arbitrage is necessary to minimise the spread between the buying and selling prices, but it *cannot* lead to the law of one price in a frictionless market. In fact, the existence of a significant amount of capital tied-up permanently in transit prevents market clearing, and thus leads to the break down of general equilibrium theory.

To sum up: markets, with many buyers and sellers, need competing arbitrageurs to minimise the spread between the buying and selling price of the same good at each date. This process will require a permanently circulating capital to ensure liquidity, and can lead to a significant spread between the buying and selling prices in the standard frictionless market.

Conclusion

The “law” of one price in frictionless markets takes its root from Aristotle’s “justice in exchange” and has dominated economic theory for centuries. This paper illustrates that this law leads to self-contradictory propositions in the valuation of companies, and in particular, in relation to dividend policy, capital structure and asset pricing. Contrary to existing beliefs, this paper proves that the present value of prospective dividends of an all-equity firm does not necessarily equal the current value of its assets even in a perfect and certain market and given full information. More generally, it identifies further hidden contradictions of the standard paradigm arising from the law of one price in frictionless markets.

This paper considers that arbitrage is necessary to minimise the spread between the buying and selling prices of any good, and for this purpose, one needs *continuous-time arbitrage permanently*. The latter requires a significant amount of *permanently circulating capital*. However, as this capital is permanently tied up in transit, it prevents market clearing. *This overlooked capital flow is as indispensable to the functioning of a market as blood circulation is to a live body, and it provides liquidity to the economy.*

To compensate an arbitrageur for the cost of capital tied-up in transit in each exchange, the selling price *must* exceed the buying price at the same date. The size of this spread can be significant particularly in risky markets. This paper brings to the fore the acute need for the reconstruction of economics on a *new* foundation to remove its internal inconsistencies.

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