Strategic Trading When Some Investors Receive Partly Information

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Abstract:

This paper models trading and price behavior in the equilibrium of a strategic trading game when some investors only receive partly information than informed trader. The model shows that the informed trader not only speculates but also manipulates the stock price. Under some condition, the informed trader may trade against his information to maintain the information superiority over market.

JEL classification: D82; D83; D84; G12; G13; G14

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1. Introduction

Every day a large number of assets change hands. Whether these assets are stocks, bonds, currencies, derivatives, real estate or just your house in the country, there are common features driving the market price of these assets. Investors base on their own information to predict the future payoff of an asset. This information affects the traders' behavior, and thus, the asset price. There are a number of important markets that exists a few very large and influential traders who have predominance on information and capital. As an example, consider the commodities futures

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markets, where large agriculture commodities firms fill the role of wealthy, well-informed traders. In Chinese stock market we name these traders “Game Banker”. Market microstructure theory and thus, our paper, name those “informed traders”. Since the existence of informed traders, lesser and late informed traders have to wrestle with at least two problems. First, they must estimate the real value of the traded asset according to their own information. Second, they have to try to infer the private information of better and early informed traders according to asset price movement. These traders are named “Traders Following Game Bankers”in the stock market of China and in our paper “partly informed traders”. There are also another class of traders in market, we name them “liquidity traders” who buy or sell shares for exogenous reasons, and insensitive to asset price. Evidently, better and early informed traders realize that their actions are being followed closely, so to keep their information predominance from being revealed to other traders, they have to alter their trading strategy.

In existing models of trading strategy, all informed investors are supposed to receive full information at the same time and, no consider the existence of partly informed traders. While these models provide important insights, they do not reflect well the reality. This paper will model trading behavior and price behavior in the equilibrium of a strategic trading game with the existence of partly informed traders.

This paper is built on the existing literature on strategic trading and manipulation. The seminal paper of Kyle (1985) investigates a model of speculative trading in which an informed insider with long-lived private information maximizes profits by exploiting strategically his monopoly power in a dynamic context. His model demonstrates how the liquidity characteristics of an efficient, frictionless market can be derived from underlying information asymmetries in a dynamic trading environment which captures some relevant features of trading in organized exchanges.

Allen and Gale (1992) consider the definition of manipulation and distinguish manipulation between trade-based, information-based and action-based. According to their classification, our model is in the class of trade based manipulation. They also present an explicit model about trade based manipulation with higher order uncertainty where all traders are price takers except for one large trader, who is either informed or uninformed. The model shows that if the large trader is uninformed, he still acts as if he had received good news. This pretense helps him manipulate the price and make profits. However, manipulation by the informed traders is not profitable. Kumar and Seppi (1992) illustrate how an informed manipulator makes profits if futures are settled by cash rather than by physical delivery. The intuition is that “cash settlement” acts as an infinitely liquid market in which pre-existing futures positions are closed out relative to the less liquid spot market. Hirshleifer, Subrahmanyam, and Titman (1994) models speculative trading in which insiders are risk averse and some of them receive information before others. However, in their model speculation would not occur without risk aversion. Bagnoli and Lipman (1996) develop a model of action-based manipulation where the manipulator pools with someone who can take an action that alters the value of the firm. In their model, the manipulator takes a position, announces a takeover bid and unwinds his position. Chakraborty and Yilmaz (1999) show that both the informed and uninformed insider can manipulate in an economy where there are a large number of rational traders, called followers, who have better information than the market about and noise traders and as a result seek to follow the insider’s trades.

In recent years, there is another branch of literature looking at the possibility of manipulation induced by the mandatory disclosure rule for insider trading activities. Fishman and Hagerty (1995) show a one-period equilibrium model of profitable manipulation when an uninformed insider successfully misleads the market into thinking he is informed. The profitable opportunity arises due to the mandatory disclosure under Rule 16a of the SEA. John and Narayanan (1997) show that with that rule, informed traders still can manipulate the price if good and bad news do not take place with equal probability. Lately, Huddart, Hughes and Levine (2000) introduce the mandatory disclosure rule to Kyle's (1985) framework and finds that insiders would apply a mixed strategy to preserve their information advantages for the future.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 and 4 characterizes the linear equilibrium. Section 5 and 6 analyzes trading behavior and price behavior in the equilibrium. Section 7 concludes the paper.

2. Model Setup

Suppose there are two assets in the market: a risky stock and a risky bond. For simplify we normalize the interest rate of the bond to zero. All market participants, viz. market maker, informed traders, partly informed traders, and liquidity traders, are risk neutral. The market maker knows the aggregate order flow before setting prices, but he does not observe the individual orders. His expected profit is zero. The above assumptions are standard for Kyle-style models.

There are 2 round of trading; this allows us to consider dynamic informed trading strategies in the simplest manner. Every round of trading includes 2 steps. First, traders submit order to buy or sell to market maker. Second, the market maker who observes the aggregate order flow sets the price equal to the expected value of the asset, conditioned on the history of orders received up to that time and trades the quantity necessary to clear the market. Since partly informed traders have no information in period 1, only informed traders and liquidity traders trade in market. In period 1, informed traders' order flow is x_1, liquidity traders' order flow is u_1, thus aggregate order flow is $X_1 = x_1 + u_1$. In period 2, partly informed traders obtain some information. Even though these informations are not exactly correct, they will participate in market. So in period 2, informed traders' order flow is x_2, partly informed traders' order flow is y, liquidity traders' order flow is u_2, and aggregate order flow is $X_2 = x_2 + y + u_2$. The trading sequence is summarized in the fig. 1.

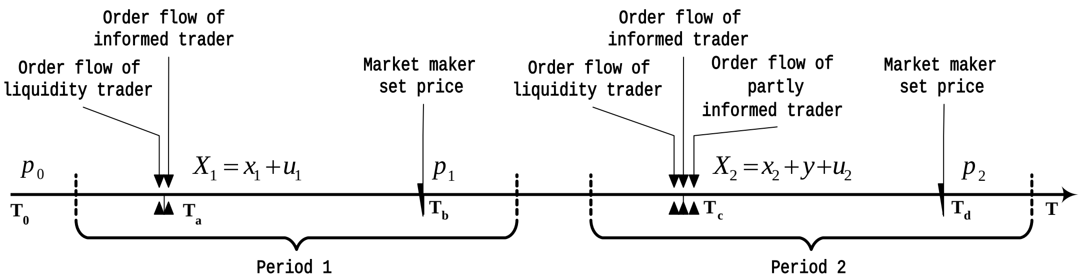

Fig.1 trading sequence

Before period 1, the true value of the stock v is known to informed trader, and will remain in the two round of trading. And the market maker knows the value of stock before period 1 is normally distributed with mean p_0, $\tilde{v}_0 \sim N(p_0, \sigma_{v0}^2)$ (In this paper, a tilde is used to distinguish a random variable from its realization); after period 1 and before period 2, $\tilde{v}_1 \sim N(p_1, \sigma_{v1}^2)$; also after period 2, $\tilde{v}_2 \sim N(p_2, \sigma_{v2}^2)$. Partly informed traders have ho information in period 1, so they do not trade; and will participate in market with partial information in period 2. But this class of traders is numerous, no cooperation among them and, lack of capital, so they cannot make optimal decision. As result, we suppose their order flow y is normally distributed $\tilde{y} \sim N(y_0, \sigma_y^2)$. Generally, informed trader has some anticipation on partly informed traders' behavior in period 2 when they trade in period 1. So we suppose that before period 1 informed trader has known the mean and variance of normal distribution of partly informed traders' order flow in period 2. The market maker has known the aggregate order flow in period 1 before setting the efficient price for period 1, so he should make some anticipation on partly informed traders' behavior. For this reason, we suppose that the market maker take this anticipation into account when he set the price for period 1. Liquidity traders buy of sell stock for exogenous reasons, we assume that their order flows u_1 and u_2 in period 1 and 2 are normally distributed $\tilde{u}_1, \tilde{u}_2 \sim N(0, \sigma_u^2)$, and independent each other. The information structure is summarized in the following table.

	Before trade	Period 1		Period 2	
	T_0	**T_a**	**T_b**	**T_c**	**T_d**
Market Maker	p_0, $\tilde{v}_0 \sim N(p_0, \sigma_{v0}^2)$ $\tilde{u}_1, \tilde{u}_2 \sim N(0, \sigma_u^2)$	**Same as left**	**Add** X_1, p_1, $\tilde{y} \sim N(y_0, \sigma_y^2)$ $\tilde{v}_1 \sim N(p_1, \sigma_{v1}^2)$	**Same as left**	**Add** X_2, p_2, $\tilde{v}_2 \sim N(p_2, \sigma_{v1}^2)$
Informed Trader	v, p_0, $\tilde{y} \sim N(y_0, \sigma_y^2)$ $\tilde{u}_1, \tilde{u}_2 \sim N(0, \sigma_u^2)$	**Same as left**	**Add** p_1	**Same as left**	**Add** p_2

Table 1 information structure

Definition 1 A Perfect Bayesian Nash Equilibrium of the trading game is defined as a strategy profile $\{x_1^*(.), x_2^*(.)\}$ and a price system $\{p_1^*(.), p_2^*(.)\}$ such that the following conditions hold:

(1)Profit maximization

$$\begin{aligned} x_2^* &\in \arg\max_{x_2} \; x_2(v - p_2) \\ x_1^* &\in \arg\max_{x_1} \; x_1(v - p_1) + x_2^*(v - p_2) \end{aligned} \tag{1}$$

(2)Market Efficiency

$$\begin{aligned} p_1^* &= E(\tilde{v} \mid X_1^*) \\ p_2^* &= E(\tilde{v} \mid X_1^*, X_1^*) \end{aligned} \tag{2}$$

This equilibrium concept is based on a dynamic programming argument. The strategy of informed trader in period 2 is required to be optimal, not only when informed trader plays his optimal strategy in period 1, but also when he plays any arbitrary strategy in period 1. In this definition, the "market microstructure" is based on a series of rules used by the market maker to determine the prices. These rules play a crucial rule because it's through them that traders affect the prices.

3 Informed Trader's Strategy under Linear Equilibrium

To derive the linear equilibrium, we begin by assuming that the prices set by market maker are linear functions of the net order flow, and have the following forms:

$$p_1 = p_0 + \lambda_1 X_1 \tag{3}$$

$$p_2 = p_1 + \lambda_2 X_2 \tag{4}$$

As in Kyle (1984), λ represents how the market maker learns about the liquidation value of the stock from the order flows. The reciprocal of λ is interpreted as market depth who refers to the ability of the market to absorb quantity without having a large effect on price.

Lemma 2 Within the framework of above information structure, the optimal order flow of informed trader in period 1 is

$$x_1 = \frac{(\lambda_1 - 2\lambda_2)(v - p_0) - \lambda_1 \lambda_2 y_0}{\lambda_1(\lambda_1 - 4\lambda_2)} \tag{5}$$

and in period 2 is

$$x_2 = \frac{v - p_1}{2\lambda_2} - \frac{y_0}{2} \tag{6}$$

if the condition

$$0 < \lambda_1 < 4\lambda_2 \tag{7}$$

is satisfied.

Proof: To derive the optimal strategy of informed trader, we use the backward induction method, because the Perfect Bayesian Nash equilibrium requires equilibrium strategies to be optimal for each information set under the given Bayesian rational belief system. In period 2, informed trader's object is to maximize his expected profit given his information.

$$\max_{x_2} E\{x_2(v - p_2)\} \tag{8}$$

$$= \max_{x_2} E\{x_2[v - p_1 - \lambda_2(x_2 + \tilde{y} + \tilde{u}_2)]\} \tag{9}$$

$$= \max_{x_2} \{x_2[v - p_1 - \lambda_2(x_2 + y_0 + 0)]\} \tag{10}$$

The solution is $x_2 = \frac{v - p_1}{2\lambda_2} - \frac{y_0}{2} \quad (\lambda_2 > 0)$ (11)

Substituting Eq.(11) into Eq.(10), we get the informed trader's profit in period 2

$$v_2(p_1) = \frac{1}{4\lambda_2}(v - p_1 - \lambda_2 y_0)^2 \tag{12}$$

Knowing the optimal order flow of informed trader in period 2, we can use again the backward induction method to derive the optimal order flow of informed trader in period 1. In period 1, informed trade faces the same problem of maximization of his expected profit:

$$\max_{x_1} E\{x_1(v - p_1) + v_2(p_1)\} \tag{13}$$

$$= \max_{x_1} E\{x_1[v - p_0 - \lambda_1(x_1 + \tilde{u}_1)] + \frac{1}{4\lambda_2}[v - p_0 - \lambda_1(x_1 + \tilde{u}_1) - \lambda_2 y_0]^2\} \tag{14}$$

$$= \max_{x_1} \{x_1[v - p_0 - \lambda_1 x_1] + \frac{1}{4\lambda_2}[v - p_0 - \lambda_1 x_1 - \lambda_2 y_0]^2 + \frac{\lambda_1^2}{4\lambda_2}\sigma_u\} \tag{15}$$

First order condition is:

$$v - p_0 - 2\lambda_1 x_1 - \frac{\lambda_1}{2\lambda_2}(v - p_0 - \lambda_1 x_1 - \lambda_2 y_0) = 0 \tag{16}$$

The solution is

$$x_1 = \frac{(\lambda_1 - 2\lambda_2)(v - p_0) - \lambda_1\lambda_2 y_0}{\lambda_1(\lambda_1 - 4\lambda_2)} \tag{17}$$

Second order condition is:

$$0 < \lambda_1 < 4\lambda_2 \tag{18}$$

The lemma 2 is proved.

Lemma 2 indicates that, informed trader not only takes advantage of his information predominance, but also considers market maker's pricing strategy and partly informed traders' order flow, to determine his trading strategy.

4 Market Maker's Pricing Strategy in Linear Equilibrium

Proposition 3 Within the framework of above information structure, market maker's optimal pricing strategy is:

$$\lambda_1 = \frac{(\lambda_1 - 2\lambda_2)(\lambda_1{}^2 - 4\lambda_1\lambda_2)\sigma_{v0}^2}{(\lambda_1 - 2\lambda_2)^2\sigma_{v0}^2 + (\lambda_1{}^2 - 4\lambda_1\lambda_2)^2\sigma_u^2} \tag{19}$$

$$\lambda_2 = \frac{\dfrac{\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2\sigma_{v0}^2}{2\lambda_2\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2 + 2\lambda_2(\lambda_1 - 2\lambda_2)^2\sigma_{v0}^2}}{\dfrac{\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2\sigma_{v0}^2}{4\lambda_2^2\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2 + 4\lambda_2^2(\lambda_1 - 2\lambda_2)^2\sigma_{v0}^2} + \sigma_y^2 + \sigma_u^2} \tag{20}$$

if the condition

$$0 < \lambda_1 < 4\lambda_2 \tag{21}$$

is satisfied.

Proof: Substituting Eq.(2) into Eq.(3), we get

$$p_1 = E(\tilde{v}_0 \mid X_1) = p_0 + \lambda_1 X_1 \tag{22}$$

and

$$p_1 - p_0 = \lambda_1 X_1 = E(\tilde{v}_0 - p_0 \mid X_1) = \frac{\mathrm{cov}(\tilde{v}_0 - p_0, \tilde{X}_1)}{\mathrm{var}(\tilde{X}_1)} X_1 \tag{23}$$

we get

$$\lambda_1 = \frac{\mathrm{cov}(\tilde{v}_0 - p_0, \tilde{X}_1)}{\mathrm{var}(\tilde{X}_1)} \tag{24}$$

According lemma 2,

$$X_1 = x_1 + u_1 = \frac{(\lambda_1 - 2\lambda_2)(v - p_0) - \lambda_1\lambda_2 y_0}{\lambda_1(\lambda_1 - 4\lambda_2)} + u_1 \tag{25}$$

But the market maker doesn't know the real value of stock v, he only observe the distribution of stock value $\tilde{v}$, thus

$$\tilde{X}_1 = \frac{(\lambda_1 - 2\lambda_2)(\tilde{v}_0 - p_0) - \lambda_1\lambda_2 y_0}{\lambda_1(\lambda_1 - 4\lambda_2)} + \tilde{u}_1 \tag{26}$$

This implies

$$\mathrm{cov}(\tilde{v}_0 - p_0, \tilde{X}_1) = \mathrm{cov}(\tilde{v}_0, \frac{\lambda_1 - 2\lambda_2}{\lambda_1{}^2 - 4\lambda_1\lambda_2}\tilde{v}_0) = \frac{\lambda_1 - 2\lambda_2}{\lambda_1{}^2 - 4\lambda_1\lambda_2}\sigma_{v0}^2 \tag{27}$$

$$\mathrm{var}(\tilde{X}_1) = (\frac{\lambda_1 - 2\lambda_2}{\lambda_1{}^2 - 4\lambda_1\lambda_2})^2\sigma_{v0}^2 + \sigma_u^2 \tag{28}$$

we get

$$\lambda_1 = \frac{(\lambda_1 - 2\lambda_2)(\lambda_1{}^2 - 4\lambda_1\lambda_2)\sigma_{v0}^2}{(\lambda_1 - 2\lambda_2)^2\sigma_{v0}^2 + (\lambda_1{}^2 - 4\lambda_1\lambda_2)^2\sigma_u^2} \tag{29}$$

Similarly, in period 2,

$$\lambda_2 = \frac{\mathrm{cov}(\tilde{v}_1 - p_1, \tilde{X}_2)}{\mathrm{var}(\tilde{X}_2)} \tag{30}$$

Since

$$\tilde{X}_2 = \frac{\tilde{v}_1 - p_1}{2\lambda_2} - \frac{y_0}{2} + \tilde{y} + \tilde{u}_2 \tag{31}$$

this implies

$$\mathrm{cov}(\tilde{v}_1 - p_1, \tilde{X}_2) = \mathrm{cov}(\tilde{v}_1, \frac{1}{2\lambda_2}\tilde{v}_1) = \frac{1}{2\lambda_2}\sigma_{v1}^2 \tag{32}$$

$$\mathrm{var}(\tilde{X}_1) = (\frac{1}{2\lambda_2})^2\sigma_{v1}^2 + \sigma_y^2 + \sigma_u^2 \tag{33}$$

We have

$$\lambda_2 = \frac{\frac{1}{2\lambda_2}\sigma_{v1}^2}{(\frac{1}{2\lambda_2})^2\sigma_{v1}^2 + \sigma_y^2 + \sigma_u^2} \tag{34}$$

But here σ_{v1}^2 is unknown, we need get its expression. Transform Eq.(25),

$$Z \equiv \frac{\lambda_1(\lambda_1 - 4\lambda_2)}{\lambda_1 - 2\lambda_2}X_1 + \frac{\lambda_1\lambda_2}{\lambda_1 - 2\lambda_2}y_0 + p_0 = v + \frac{\lambda_1(\lambda_1 - 4\lambda_2)}{\lambda_1 - 2\lambda_2}u_1 \tag{35}$$

Notice that $\tilde{Z} \sim N(v, \frac{\lambda_1^2(\lambda_1 - 4\lambda_2)^2}{(\lambda_1 - 2\lambda_2)^2}\sigma_u^2)$ is a transform of observed order flow, and its mean is real value of asset. According appendix, we have

$$\sigma_{v1}^2 = (\frac{1}{\sigma_{v0}^2} + \frac{(\lambda_1 - 2\lambda_2)^2}{\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2})^{-1} = \frac{\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2\sigma_{v0}^2}{\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2 + (\lambda_1 - 2\lambda_2)^2\sigma_{v0}^2} \tag{36}$$

Substituting Eq.(36) into Eq.(34), we get

$$\lambda_2 = \frac{\dfrac{\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2\sigma_{v0}^2}{2\lambda_2\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2 + 2\lambda_2(\lambda_1 - 2\lambda_2)^2\sigma_{v0}^2}}{\dfrac{\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2\sigma_{v0}^2}{4\lambda_2^2\lambda_1^2(\lambda_1 - 4\lambda_2)^2\sigma_u^2 + 4\lambda_2^2(\lambda_1 - 2\lambda_2)^2\sigma_{v0}^2} + \sigma_y^2 + \sigma_u^2} \tag{37}$$

Proposition 3 is proved.

5 Trade-Based Manipulation

Proposition 4 If informed trader knows liquidation value of stock and, liquidation value is equal to initial price, $v = p_0$, then the informed trader's profit function based on market manipulation is:

$$\frac{1}{4}\lambda_2^2 y_0^2 (1 - \frac{\lambda_1}{\lambda_1 - 4\lambda_2}) + \frac{\lambda_1^2}{4\lambda_2}\sigma_u \tag{38}$$

Informed trader's optimal order flow in period 1 is:

$$x_1 = \frac{-\lambda_2 y_0}{\lambda_1 - 4\lambda_2} \tag{39}$$

in period 2 is:

$$x_2 = \frac{p_0 - p_1}{2\lambda_2} - \frac{y_0}{2} \tag{40}$$

if the condition

$$0 < \lambda_1 < 4\lambda_2 \tag{41}$$

is satisfied.

proof: Since $v = p_0$, simplify Eq.(5)as follows

$$x_1 = \frac{-\lambda_2 y_0}{\lambda_1 - 4\lambda_2} \tag{42}$$

Similarly Eq.(6)becomes

$$x_2 = \frac{p_0 - p_1}{2\lambda_2} - \frac{y_0}{2} \tag{43}$$

Substituting $v = p_0$ and Eq.(40) into Eq.(15), simplifying, we get the informed trader's profit function based on market manipulation

$$\frac{1}{4}\lambda_2^2 y_0^2 (1 - \frac{\lambda_1}{\lambda_1 - 4\lambda_2}) + \frac{\lambda_1^2}{4\lambda_2}\sigma_u \tag{44}$$

Proposition 4 is proved.

Analyzing Eq.(39) and Eq.(40), we find that the direction of x_1 is same as y_0, and different from x_2. So proposition 4 indicates one situation in which informed trader knows or schemes one event that will happen in future, although this event doesn't influence liquidation value of stock, it'll affects partly informed traders' behavior. Informed trader gains the profit by buying (selling) the stock before partly informed traders and, selling (buying) the stock when partly informed traders enter the market.

Eq.(38)shows also an interesting phenomenon: even though with absence of partly informed traders, $y_0 = 0$, informed trader's profit function is positive, $\frac{\lambda_1^2}{4\lambda_2}\sigma_u$. Why? Because informed trader knows the liquation value of stock, he can gain profit through the stock price volatility caused by liquidation traders.

6 Trading against information

In Kyle (1985) the information monopolist retrains his trading activities in order to protect information for future trading rounds. In contrast, informed trader in our paper could trade more aggressively in period 1. Under some circumstance, in period1, informed trader may manipulate the price by trading against his private information to move the price in his favor.

Proposition 5 Within the framework of above information structure, if the condition

$$(\lambda_1 - 2\lambda_2)(v - p_0)^2 > \lambda_1\lambda_2 y_0 (v - p_0) \tag{45}$$

is satisfied, and

$$0 < \lambda_1 < 4\lambda_2 \tag{46}$$

informed trader will trade against his private information in period 1.

Proof: Trading against his private information means

$$x_1(v - p_0) = [\frac{(\lambda_1 - 2\lambda_2)(v - p_0) - \lambda_1\lambda_2 y_0}{\lambda_1(\lambda_1 - 4\lambda_2)}](v - p_0) < 0 \tag{47}$$

This implies

$$(\lambda_1 - 2\lambda_2)(v - p_0)^2 > \lambda_1\lambda_2 y_0 (v - p_0) \tag{48}$$

Proposition 5 is proved.

The result of proposition 5 is too complex to understand, so we give the following deduction in order to better realize.

Deduction 6 If the condition Eq.(45) is more strict

$$0 < \lambda_1 < 2\lambda_2 \tag{49}$$

then informed trader manipulate the price by trading against his private information in period 1 when the following conditions hold:

1) if $v - p_0 > 0$ (50)

$$v - p_0 < \frac{\lambda_1\lambda_2}{\lambda_1 - 2\lambda_2} y_0 \tag{51}$$

2) if $v - p_0 < 0$ (52)

$$v - p_0 > \frac{\lambda_1\lambda_2}{\lambda_1 - 2\lambda_2} y_0 \tag{53}$$

Proof: Since $\lambda_1 - 2\lambda_2 < 0$, Eq.(45) leads to

$$(v - p_0)^2 > \frac{\lambda_1\lambda_2}{\lambda_1 - 2\lambda_2} y_0 (v - p_0) \tag{54}$$

Distinguish two cases according to $(v - p_0)$ sign, we get immediately the above deduction.

The result of Deduction 6 is more comprehensible, because two cases correspond to the real market operations. When $v - p_0 > 0$, since $\frac{\lambda_1\lambda_2}{\lambda_1 - 2\lambda_2} < 0$, in order to satisfy Eq.(49), we get

$y_0 < 0$ and, the absolute value of y_0 should be enough large. The corresponding market real operation is just the same. Informed trader know that the liquidation value of stock is higher than current price, but if he predicts that partly informed traders will sell the stock in period 2 because of their partly information and this amount is large enough, informed trader will sell the stock in period 1 to maximize the total profit in two periods by increasing the gap between the liquidation value of stock and its current price. The case $v - p_0 < 0$ indicates the inverse situation and the analysis is just the same. We don't discuss it here.

7 Conclusion

The existence of partly informed trader leads to the great change of all market participants' trading strategy. Our paper models this change. The model indicates that, informed trader not only takes advantage of his information predominance, but also considers partly informed traders' behavior, to determine his trading strategy. In order to maximize his profit, informed trader speculates and manipulates the stock and, under certain circumstance he trades against his private information in first period.

Appendix

Suppose that the density of probabilities a priori of random variable μ is $g(\mu)$. The observed values of random variable x are independently and identically distributed, their density is $f(x \mid \mu)$ when given μ. Thus, the density of probabilities a posteriori of μ is:

$$g(x \mid \mu) = \frac{g(\mu) f(x \mid \mu)}{\int f(x \mid \mu) g(\mu) d\mu} \tag{55}$$

We assume that random variable μ is normally distributed, $N(m, \sigma_\mu^2)$. Given the value of random variable μ, observed values x are normally distributed, $N(\mu, \sigma_x^2)$. This implies:

$$g(\mu) = \frac{1}{\sqrt{2\pi}\sigma_\mu} \exp[-\frac{1}{2\sigma_\mu^2}(\mu - m)^2] \tag{56}$$

$$f(x \mid \mu) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp[-\frac{1}{2\sigma_x^2}(x - \mu)^2] \tag{57}$$

According to Eq.(55), we get the density of probabilities a posteriori of μ:

$$\left(\frac{\sigma_x^2 + \sigma_\mu^2}{2\pi\sigma_x^2\sigma_\mu^2}\right)^{\frac{1}{2}} \exp\left[-\left(\frac{\sigma_x^2 + \sigma_\mu^2}{2\sigma_x^2\sigma_\mu^2}\right)\left(\mu - \frac{\frac{m}{\sigma_\mu^2} + \frac{x}{\sigma_x^2}}{\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_x^2}}\right)^2\right] \tag{58}$$

Thus, the posteriori distribution of μ when given the observed value x is:

$$N\left[\frac{\frac{m}{\sigma_\mu^2} + \frac{x}{\sigma_x^2}}{\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_x^2}}, (\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_x^2})^{-1}\right] \tag{59}$$

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