

Paper presented at

Alternative Perspectives on Finance

Fifth Biennial Conference
Dundee, Scotland

July 23 – 25, 2000

For further information visit our website

<http://www.departments.bucknell.edu/management/apfa/>

CAPITAL BUDGETING AND THE
REINVESTMENT ASSUMPTION

By Walter J. Reinhart

For Alternative Perspective on Finance
Fifth Biennial Conference
July 23 – 25, 2000

Walter J. Reinhart
Finance Professor
Loyola College
4501 North Charles Street
Baltimore, MD 21210

Tel: 410.617.2656
Fax: 410.617.5035
Email: Reinhart@Loyola.edu

CAPITAL BUDGETING AND THE REINVESTMENT ASSUMPTION

Abstract

One of the most misunderstood concepts in the fields of finance and accounting is the “reinvestment assumption” in investment decisions. This assumption is used in investment decisions involving time value of money concepts and range from capital projects to bonds and stock selection. It is the purpose of this paper to explore the reinvestment assumption in net present value and internal rate of return techniques by a mathematical exploration, including a sample problem, and to discuss some inferences involved. This exploration shows that the need for reinvestment is a false assumption. The paper also briefly explores the implications of this finding for investment decisions.

Introduction

One of the most misunderstood concepts in the fields of finance and accounting is the “reinvestment assumption” in investment decisions. This assumption is used in investment decisions involving time value of money concepts and range from capital projects to bonds and stock selection. Capital budgeting, where it is generally assumed that the time value techniques of net present value (NPV) and/or internal rate of return (IRR) are the appropriate criterion to employ, is the focus of this paper. Financial literature has long recognized that NPV and IRR can give conflicting rankings for the same set of investment alternatives. The source of this conflict is almost universally traced to a “reinvestment assumption” that is considered implicit in all of the present value techniques. For example one author [13] states “The ultimate factor that causes the difference in rankings is the implicit reinvestment rate assumptions incorporated into the NPV and IRR formulas.” Another author [12] says “The outcome depends on what assumptions the decision maker chooses to make about the implied reinvestment rate for the net cash flows generated from each project.” Still another author [2], among many, presents the following: “*The NPV method implicitly assumes that the rate at which cash flows can be reinvested is the cost of capital, whereas the IRR method implies that the firm has the opportunity to reinvest at the IRR. These assumptions are inherent in the mathematics of the discounting process.*” Periodically the assumption of reinvestment of intermediate flows at a single rate of discount comes under investigation. Even though Dudley (1972) looked at this

assumption correctly errors still abound in the literature and in many basic finance [1, 3, 6, 7, 10, 11, 15, 16] and accounting [8, 9] textbooks.

The source of confusion regarding reinvestment apparently stems from an incomplete parenthetical summary of earlier work and the assumption that compounding is an inherent property of any form incorporating the principal of time-value of money – whether discounting or compounding. Unfortunately the incomplete summary has found its way into many basic textbooks with the result that many users of NPV and IRR falsely assume reinvestment is necessary for the discounting techniques to be valid [14]. By itself, this partial summary in the literature most likely would not be enough for the error to continue. Rather, the inherent assumption of compounding is also required: for compound (future value) the investment/reinvestment of intermediate flows after they are received is, obviously, required. However, in this compounding process, it is not necessary to assume an investment rate before these flows occur. Instead, the rate at which one assumes any future intermittent cash flow is invested at determines the overall future value. On the other hand, in discounting, which is the opposite of compounding, if anything, we need to assume an investment rate before the intermediate flows occur, not after. In essence, we need to assume that the initial outlay is being invested at some rate of return. What is done with the future intermittent flows after they are received does not influence present value. It should be recognized that discounting and compounding are not reciprocals unless we have single flows, and then reinvestment is a mute question. It is the purpose of this paper to explore the reinvestment assumption in NPV and IRR by a mathematical exploration, including a sample problem, and to discuss some inferences involved. The final section explores the implications for investment decisions.

Methodological Approach

An exploration by a mathematical analysis suggests that the general definition of the discount techniques should be stated and then modified to (1) explicitly account for reinvestment of cash flows, and (2) to allow for various rates during different time periods. For simplicity, we choose a three period model to highlight our analysis.

The net present value (NPV) of an investment proposal is equal to the sum of all future flows (CF_t) generated by the project discounted at the required return (r) (“cost of capital”) to the present, less the initial cost of the project (CF_0):

$$NPV = \sum_{t=1}^n \frac{CF_t}{(1+r)^t} - CF_0 \quad (1)$$

The internal rate of return (k) is defined as the discount rate which equates the present value of the future flows of the project to the initial cost of the investment in present value terms, or where the net present value is zero:

$$NPV = 0 = \sum_{t=1}^n \frac{CF_t}{(1+k)^t} - CF_0 \quad (2)$$

or:

$$CF_0 = \sum_{t=1}^n \frac{CF_t}{(1+k)^t} \quad (2a)$$

Given the fact that IRR is defined as the rate where NPV equals zero there is little doubt that we are talking about present value and not future value when we use either technique to evaluate an investment proposal. The next step is to recognize that IRR is indeed just a point solution where NPV is equal to zero. Then considering the possible problems (e.g., multiple rates) that may arise with IRR, and for ease of exposition and without loss of generality this paper discusses only the present value technique. Moreover, since IRR is just a special case of NPV any conclusion regarding NPV is readily extended to IRR.

Having established the base relationship, equation (1) can now be rewritten in future value (FV) terms to show any “reinvestment assumption” by multiplying the equation through by the factor $(1+r)^n$:

$$NPV (1+r)^n = \sum_{t=1}^n CF_t (1+r)^{n-t} - CF_0 (1+r)^n = FV \quad (3)$$

Expanding the above to show three periods, equation (4) presented below appears to demonstrate that if a net investment of CF_0 dollars today is to earn a unique compound rate of return, r , to generate a net present value of NPV, the cash throw-offs must be reinvested at that same rate. It

is unique in the sense that the future value (including all intermittent flows) and present value can be related by a single discount/compounding rate.

$$NPV(1+r)^3 = CF_1(1+r)^2 + CF_2(1+r) + CF_3 + P_3 - CF_0(1+r)^3 \quad (4)$$

where: P_3 is a third period flow and represents a salvage or maturity value

Leaving the above in unsimplified form and solving for NPV yields:

$$NPV = \frac{CF_1(1+r)^2}{(1+r)^3} + \frac{CF_2(1+r)}{(1+r)^3} + \frac{CF_3 + P_3}{(1+r)^3} - \frac{CF_0(1+r)^3}{(1+r)^3} \quad (5)$$

which is an alternative form of (1) but with the alleged implied reinvestment assumption made explicit. Expanding (5) to include for varying investment rates (r_1, r_2, r_3) in each time period results in the following:

$$NPV = \frac{CF_1(1+r_2)(1+r_3)}{(1+r_1)(1+r_2)(1+r_3)} + \frac{CF_2(1+r_3)}{(1+r_1)(1+r_2)(1+r_3)} + \frac{CF_3 + P_3}{(1+r_1)(1+r_2)(1+r_3)} - \frac{CF_0(1+r_1)(1+r_2)(1+r_3)}{(1+r_1)(1+r_2)(1+r_3)} \quad (6)$$

Simplifying this equation shows that for present value calculations the forward rates cancel out, regardless of whether they are constant or different for each future period. Since we are dealing with predicting the future flows and discount rates it would not be logical to say that a flow with given risk characteristics should be compounded forward at one rate and then discounted back at another rate. These results are presented in equation (7):

$$NPV = \frac{CF_1}{(1+r_1)} + \frac{CF_2}{(1+r_1)(1+r_2)} + \frac{CF_3 + P_3}{(1+r_1)(1+r_2)(1+r_3)} - CF_0 \quad (7)$$

As shown, the forward cash flows are not required to be (re)invested. They can be invested at any rate or consumed and the present value will not change. It should be noted that equation (7) is identical to equation (1) except for the fact that the discounting process is delineated. If the flows are invested, and compounded at some expected return, given an ex anti risk/return relationship, then they would have to be discounted back by the same rate to the period in which they originate. Thus, in essence they are only discounted back from their point of origination to present value by the required rate of the initial investment. This is true since it is not logical nor

consistent to assume, ex ante, that the investment and discount rates for a given flow would not be equal for each future period. To compound flows out at one rate of return and then discount them back at another rate of return would imply that market imperfections exist to generate “excess return” for a given risk level and that it is not necessary to account for that level of risk. Hence, for present value calculations they must cancel out; and, as shown in equations (6) and (7), this is the case even if the discount rate is not the same for each time period.

A simplified practical example to demonstrate that cash flows do not have to be reinvested is to examine a savings account which pays, say, a nine percent (9%) rate of return compounded annually and the initial outlay is \$1,000. The interest payment of \$90 per year can either be reinvested with the principal amount and earn 9% per annum, or sent to the investor. Placing the interest payments and principal flows in a three-year time continuum shows the following cash flow pattern:

Time	0	1	2	3
Flow	-1000	+90	+90	+90 +1000

It is obvious that the IRR=9% ($PV = 0$ at 9%), and that the investment is acceptable if the required return is equal to or less than 9%. Exploring the two options associated with interest payments allows us to determine how the present value [PV] and future value [FV] are affected when the intermediate flows are invested or consumed. In the following mathematical analysis the \$1000 initial cost is not included for ease of presentation. First, assume reinvestment at 9%:

$$PV = \sum_{t=1}^3 \frac{90(1+.09)^{n-t}}{(1+.09)^t(1+.09)^{n-t}} + \frac{1000}{(1+.09)^3}$$

$$PV = \frac{90(1+.09)(1+.09)}{(1+.09)(1+.09)(1+.09)} + \frac{90(1+.09)}{(1+.09)^2(1+.09)} + \frac{90}{(1+.09)^3} + \frac{1,000}{(1+.09)^3}$$

$$PV = \$1,000$$

Including the initial cost of \$1000 results in, as expected, an NPV equal to zero. Turning our attention to future value, at 9% reinvestment we find the following:

$$FV = \sum_{t=1}^3 90 (1 + .09)^{3-t} + 1000 (1 + .09)^{3-3}$$

$$FV = 90(1+.09)^2 + 90(1.09)^1 + 90(1.09)^0 + 1000(1+.09)^0$$

$$FV = \$1,295$$

The future value when investing the intermittent flows is equal to \$1295. Examining the second alternative where we assume the receipt and consumption of the interest payments shows the following:

$$PV = \sum_{t=1}^3 \frac{90}{(1 + .09)^t} + \frac{1000}{(1 + .09)^3}$$

$$PV = \frac{90}{(1 + .09)} + \frac{90}{(1 + .09)^2} + \frac{90}{(1 + .09)^3} + \frac{1000}{(1 + .09)^3}$$

$$PV = \$1000$$

Again, including the initial cost of \$1000 results in an NPV of zero. For the future value, with consumption, we find:

$$FV = \sum_{t=1}^3 90 (1 + (-1.0))^{3-t} + 1000 (1 + (-1.0))^{3-3}$$

$$FV = 90(1-1)^2 + 90(1-1)^1 + 90(1-1)^0 + 1000(1-1)^0$$

$$FV = \$1,090$$

In this second case the future value is the amount received in the last year, \$1090.

Analyzing the preceding example shows that the present value is \$1,000 (obviously) and the NPV is zero regardless of whether intermediate flows are reinvested or consumed. However, the future values differ, \$1,295 versus \$1,090. From this, it may be concluded that reinvestment of intermittent flows is not necessary for present value calculations, while it is necessary for future values. However, even for present value calculations a “reinvestment assumption” of sorts is required – we must assume that the initial investment is continuously invested to generate the future flows.

Implications

The preceding discussion has been limited to the question and confusion about reinvestment in investment decisions. It was shown that it is not necessary to assume reinvestment of cash throw-offs in the determination of present value. By simple extension since IRR is just a special case of net present value we can conclude that the internal rate of return method does not require a reinvestment assumption regarding future flows generated by the project. Rather, the initial outlay is assumed to be “reinvested” over the life of the investment, and the flows can be invested or consumed. These findings raise some implications regarding investments and the criterion employed to determine which are the better alternatives.

The criteria of net present value and internal rate of return under certain conditions, can give conflicting recommendations as to the desirability of competing/mutually-exclusive investments. As noted previously, the difference in signals is often attribute to different reinvestment assumptions. However, if no assumption is required, unless future values are calculated, the preceding statement must be questioned. Conflicting decisions are more properly attributed to a difference in the magnitude of the initial outlay or to different cash flow patterns (including different life spans).

A reinvestment rate becomes important when investments of differing maturities, or maturities of other than one's time horizon, are being evaluated. In these cases, future value may be a more appropriate indicator than present value. If future values are used, then the investment rate should be the marginal efficiency of capital in each of the future periods, given the necessary risk-return relationships. Assuming reinvestment at a constant or unique rate implies that the yield curve must be flat and history indicates that generally yield curves are not flat.

A variation of these capital budgeting techniques is the modified internal rate of return [MIRR] which actually shows a reinvestment rate in its formulation. Generally it is assumed that the future positive intermittent flows are compounded at the required return (“cost of capital”) to generate an artificial terminal value. A discount rate is then determined which equates this single terminal flow to the initial cost of the project. This discount rate is called the modified internal rate of return. However, as alluded to above, the inconsistency of compounding at one rate of

return and discounting at another rate in an *ex anti* evaluation represents a serious methodological flaw in the technique.

Regarding the appropriate techniques to employ in investment decisions, it is generally concluded that present value is more appropriate unless capital is rationed. In this latter case, IRR, or a benefit cost index, or a sophisticated programming solution should be employed. To arrive at a definitive answer, additional research is required and would be the subject of another paper. However, recognizing the fact that present value techniques such as NPV and IRR do not require a reinvestment assumption may help clear up the debate regarding the best criterion to employ when choosing between projects. On the other hand, if investment of future intermittent flows is considered important, a future value criterion needs to be developed.

REFERENCES

1. Besley, S. and Brigham, E., *Essentials of Managerial Finance*, Second Edition, Orlando, Florida: The Dryden Press, 2000.
2. Brigham, E. and Houston, J., *Fundamentals of Financial Management*, Concise Second Edition, Fort Worth, Texas: The Dryden Press, 1999.
3. Damodaran, A., *Applied Corporate Finance: A User's Manual*, New York, New York: John Wiley and Sons, Inc., 1999.
4. Downes, J. and Goodman, J., *Dictionary of Finance and Investment Terms*, Hauppauge, New York: Barron's Educational Series, Inc., 1998.
5. Dudley, Jr. C., "A Note on the Reinvestment Assumptions in Choosing Between Net Present Value and Internal Rate of Return," *The Journal of Finance* (September 1972), 85-91.
6. Emery, D., Finnerty, J., and Stowe, J., *Principles of Financial Management*, Upper Saddle River, New Jersey: Prentice-Hall, Inc. 1998.
7. Fabozzi, F., *Bond Markets, Analysis and Strategies*, Fourth Edition, Upper Saddle River, New Jersey: Prentice-Hall, Inc., 2000.
8. Garrison, R. and Noreen, E., *Managerial Accounting*, Ninth Edition, Irwin/McGraw-Hill, 2000.
9. Hilton, R., *Managerial Accounting*, Irwin/McGraw Hill, Boston, Massachusetts: 1999.
10. Jones, C., *Investments: Analysis and Management*, New York, New York: John Wiley and Sons, Inc., 2000.
11. Keown, A., Scott, Jr., D., Martin, J., and Petty, J., *Basic Financial Management*, Seventh Edition, Upper Saddle River, New Jersey: Prentice-Hall, Inc., 1996.
12. Moyer, C., McGuigan, J., and Ketlow, W., *Contemporary Financial Management*, Cincinnati, Ohio, South-Western College Publishing, 1997.
13. Pinches, G., *Financial Management*, Reading Massachusetts, Addison Wesley Educational Publishing, 1998.
14. Reinhart, W. "An Investigation of the Reinvestment Assumption in Investment Decisions," Southwest Finance Association Meeting, March, 1978.

15. Shapiro, A. and Balbirer, S., *Modern Corporate Finance: A Multidisciplinary Approach to Value Creation*, Upper Saddle River, New Jersey: Prentice-Hall, Inc., 2000.
16. VanHorne, J., *Financial Management and Policy*, Upper Saddle River, New Jersey: Prentice-Hall, Inc., 1998.