Optimal Weights for Divisia Aggregation using a Neural Network Approach

Gazely A.M and Binner J.M

Department of Finance & Business Information Systems

Nottingham Business School

Burton Street, Nottingham, NG1 4BU

England UK.

Email : alicia.gazely@ntu.ac.uk

Email : jane.binner@ntu.ac.uk

Abstract

An important element in macroeconomic policy is the relationship between money supply and inflation. The conventional way of measuring the amount of money circulating in an economy is to simply sum the various constituent liquid liabilities of commercial and savings banks. However the belief is widely established that this method of arriving at broad money aggregates is seriously flawed and based on untenable assumptions. From a micro-demand perspective it is hard to justify adding together component assets which have differing yields which vary over time, since the accepted view is that only things that are perfect substitutes can be combined as one ‘commodity’. An important alternative is the Divisia weighted index, in which the component assets are not assumed equal or invariant over time. Components are weighted on the basis of the quantity of monetary services provided by each, expressed in terms of the interest yield.

This study utilises a powerful and flexible tool, the Artificial Intelligence technique of neural networks to develop this approach. Can a new empirically weighted measure of money be constructed that more closely captures the monetary services flow provided by the component assets? Sensitivity analyses were derived from neural networks trained with UK data to forecast inflation directly from the components of Divisia, and used to construct a 'new' Divisia style aggregate with superior forecasting performance when compared with the traditional Divisia aggregate. Results to date indicate that the combination of Divisia measures of money with the artificial neural network offers a promising starting point for the development of an optimised Divisia aggregate and hence an improved model of the relationship between money supply and inflation. Our results are reinforced by a growing body of evidence from empirical studies around the world which demonstrate that weighted index number measures may be able to overcome the drawbacks of simpler measures. Ultimately, such evidence could reinstate money as a credible policy tool for controlling inflation.

Key words: Divisia money; Neural networks; Inflation forecasting;

Introduction

The traditional reason for regarding ‘money’ as critically different from other assets is that it has a direct role in transactions and hence has a direct role in the trading activity of a market economy. According to the Quantity Theory tradition of economics, the money stock will determine the general level of prices (at least in the long term) and according to the monetarists it will influence real activity, in the short run. In other words, there is a relationship between money supply and inflation, and this relationship is an important element in macroeconomic policy as governments try to control inflation.

Measuring money supply is no easy task. Component assets range from 'narrow money', which includes cash, non interest bearing demand deposits and sight deposits on which cheques can be drawn, to 'broader' money, which includes non-checkable liquid balances and other liquid financial assets, such as Certificates of Deposit. In practice, official measures have evolved over time according to policy needs. Obviously many of these assets yield an interest rate and could thus be chosen as a form of savings as well as being available for transactions. Financial innovation, in particular liberalisation and competition in banking, has led to shifts in demand between the components of 'money' which have undermined earlier empirical regularities and made it more difficult to distinguish money which is held for transactions purposes from money which is held for savings purposes (Mullineux, 1996).

Secondly, there is the question of how to combine the different components, since they are not perfect substitutes for one another. They provide different levels of monetary services for transactions (liquidity) and different yields (interest). As payments technology progresses, so the monetary services provided will change disproportionately - not only do interest rates vary with time but liquidity can be enhanced too. Despite this variation in the characteristics of assets, they are traditionally combined for use as monetary targets or indicators by simply adding them up - the 'simple sum' aggregate. This is hard to justify and there is a growing body of evidence to support alternative solutions which incorporate a weighting mechanism, notably the aggregate devised by Divisia (1925) and known as the Divisia aggregate.

Barnett (1980) has demonstrated that Divisia is the correct index to encapsulate a monetary aggregate and its concomitant price index. The construction has its roots firmly based in microeconomic aggregation theory and statistical index number theory. Barnett et al. (1992) provide a survey of the relevant literature, whilst Drake and Mullineux (1997) review the construction of Divisia indices and associated problems. The theoretical case in favour of weighted index number measures of money is convincing and difficult to dispute at the microeconomic level and the empirical case is growing. For example, Barnett and Spindt (1979) and Belongia and Chalfant (1989) showed Divisia money has a superior information content to simple sum, whilst Barnett (1980) demonstrated that shifts in demand for money functions in the USA were eliminated when Divisia replaced simple sum measures. Further support for Divisia is forthcoming from Barnett, Offenbacher and Spindt (1984) and Serletis (1988). Chrystal and MacDonald (1994) produced a substantial work encompassing results for seven countries and a broad spectrum of econometric tests. Results are not always clear cut however and overwhelmingly government policy makers continue to use the simple sum aggregate. The emerging picture is that further empirical evidence is urgently required and that the construction of the Divisia monetary index has not yet been optimised, e.g. Binner (1990), Ford and Mullineux (1996).

In the UK the guiding principle for macroeconomic policy has been direct control of inflation since departure from the ERM in September 1992. Currently the policy of UK monetary authorities is to identify leading indicators of macroeconomic conditions which will alert policy makers sufficiently early to allow the necessary action to be taken to control and remedy the problem. Owing to lags in the effect of monetary policy this requires a forecast of inflation up to 24 months ahead and the Bank of England provides a regular quarterly 'Inflation Report' which includes quantified forecasts for inflation.

This paper aims to provide further evidence that the behaviour of Divisia monetary aggregates should be taken seriously by both policy makers and academic economists. Our focus is the possibility of optimising the construction of the Divisia aggregate by substituting component weights that are empirically derived, for the traditional calculation of weights from interest rates. The tool used is the artificial neural network, and the following section explains why it is appropriate for the task.

The artificial neural network

Neural networks in hardware form are constructions made up of many relatively simple processors which are interconnected. Some elements (neurons) receive inputs from outside the system and all elements produce outputs. For data modelling purposes, neural networks are normally created by a computer program and used to store model parameters in the form of the architecture of the network. Weights are applied to values passed from one neuron to another and functions are employed to transfer inputs to outputs at individual neurons. A simple neural network is represented in Figure 1 and consists of three inputs, two middle neurons in the "hidden" layer and a single output. Initially the network has randomly allocated weights on each connection. A training process follows which consists of presenting inputs and a desired output repeatedly to the network. The weights are adjusted each time to minimise the error between the desired output and the network output. If the network has converged satisfactorily (settled with an optimal set of weights) after some number of repetitions determined by the experimenter then it is said to be trained. Unseen inputs can then be presented to the network to obtain a response (the 'testing' process).

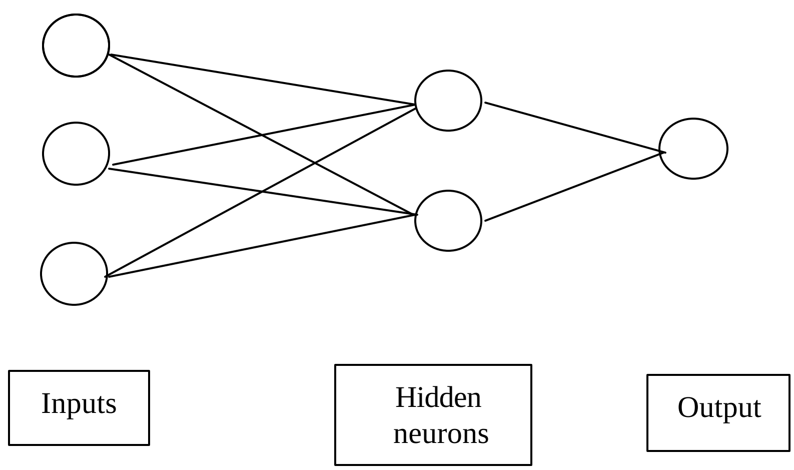

Figure 1: A simple neural network

The literature of empirical evidence which investigates the long-run relationship between money growth and inflation (see for example Drake et al., 2000) has been based largely on standard econometric methodology using cointegration techniques. The advantage of neural network technology for this problem, however, is that neural networks are inductive. Even when there is no exact knowledge of the rules determining the features of a given phenomenon, knowledge of empirical regularities can still allow the phenomenon to be modelled. The process of training permits the network to allow powerful explanatory variables full influence at the expense of weaker ones, and to model complex non-linear relationships. The technique is ideal for economic phenomena where the central task is to model a system of immense complexity without losing predictive power. Neural networks are also relatively robust to noise in data compared to traditional statistical techniques. It seems, however, that the potential of techniques such as the neural network, which are non-traditional in econometric investigations, has not been fully exploited in the area of monetary economics.

An exception is the work of Dorsey (2000) who used a genetic algorithm technique to explore the potential of the neural network approach to forecasting US inflation, without laying much emphasis on the prediction of values at the end of the time series. This philosophy has been followed in the present work. The simplest possible designs and procedures have been used throughout since the aim is to explore the potential of the technology, rather than to produce forecasts for predictive purposes. It is virtually certain that more accurate forecasting could be achieved by building more complex models.

Data and Method

The experiment used a very simple model of the relationship between money supply and inflation:

$$\Pi_t = F(M_{t-4}, \Pi_{t-1}, T_t)$$

which takes inflation in the current quarter to be a function of just three elements:

M_{t-4} money supply in the quarter one year earlier. This is represented by either just one measure (an aggregate such as the Divisia index) or five measures for the five Divisia components (see below)

Π_{t-1} inflation for the preceding quarter, an auto-regressive term

T_t time; the neural network does not explicitly take account of the sequence of values in the time series and in fact the values are presented in random order to the network during training, so this variable caters for environmental changes over the sample period which are not reflected in the money measure.

The measure of money currently of interest to the UK monetary authorities is M4. Quarterly data for the five monetary components of Divisia M4 and also published Divisia M4 were supplied directly by the Bank of England. Inflation is defined in terms of the government's target measure RPIX, which is the Retail Price Index excluding mortgage payments. The quarterly RPIX series was provided by the Central Statistical Office and constructed as year-on-year growth rates. 66 quarters of price data from 1979 Q4 to 1996 Q1 yielded a 61 quarter sample for inflation (1981 Q1 to 1996 Q1) following the calculation of year-on-year inflation and a one quarter time lag in the modelling process. Monetary components and Divisia series data from 1980 Q1 to 1996 Q1 yielded 61 quarter samples after catering for a four quarter time lag. Of these 61 quarters, 10 quarters of data (1993 Q4 to 1996 Q1) were withheld for out-of-sample forecasts leaving 51 quarters for training.

The components of Divisia can be listed as

N/C = Notes and coins
NIBD = Non-interest-bearing bank deposits
IBSD = Interest-bearing bank sight deposits
IBTD = Interest-bearing bank time-deposits
BSD = Building society deposits

For each component, there are three sectors: Households, Other Financial Corporations and Non-financial corporations. For this study, the three sectors were aggregated.

First, a network was constructed which used the five components of Divisia lagged by four quarters, together with the time trend variable and the auto-regressive term (a total of 7 inputs) to train for one output, inflation for the current quarter. The root mean squared error (RMS) provided a measure of fit for both the within-sample results (training on 51 quarters) and the out-of-sample results (testing on 10 quarters). Then a measure of the sensitivity of the output to each input was obtained from the trained network, using the facilities of the neural network software program. A higher value represents a greater sensitivity and therefore means that the component is relatively more important. These five values (which sum to 1) were then used as the weights to construct a new (very crude) Divisia. For any quarter, the new index was calculated as the sum of the product of each of the five components with the corresponding sensitivity value. A network was trained with this new variable plus the time trend variable and the auto-regressive term as before (three variables). Finally, a third network was trained similar to the second one in all respects except that the traditional Divisia index was substituted for our newly constructed version.

The experiment was carried out using the software Pattern Recognition Workbench (Unica Technologies, Inc.) The neural network architecture used for all the networks was a standard back-propagation network with either five or four hidden neurons in one layer; the number of hidden neurons had previously been determined by preliminary experiments which showed that for 7 inputs, five hidden neurons were optimal and for 3 inputs, four were optimal. Other experimental parameters were kept as standard as possible, and in any case constant for all the networks. No doubt fine-tuning could improve forecasting ability - however it was felt to be more important to keep the experiment simple as far as possible, than to minimise error.

The networks were all trained for 20,000 iterations of the data set at which point they all appeared to have settled into a stable state. It is possible that premature stopping could have improved accuracy. However this technique, used to prevent overfitting, introduces an element of chance into the proceedings and reduces repeatability. Overfitting is the result of using too many free parameters (hidden neurons) and cannot be reliably controlled by premature stopping.

Results and discussion

The sensitivity results from the first network are shown in Table 1, in order of sensitivity from highest to lowest..

IBTD = Interest-bearing bank time-deposits	0.218300
Time trend (T)	0.198421
BSD = Building society deposits	0.147345
IBSD = Interest-bearing bank sight deposits	0.139904
N/C = Notes and coins	0.126628
NIBD = Non-interest-bearing bank deposits	0.116607
Auto-regressive term (Π_{t-1})	0.052795
	1.000000

Table 1: Sensitivity of inflation to Divisia money components

Notes and coins, and non-interest bearing bank deposits are the most liquid assets. The greater the liquidity, the higher the level of monetary services provided and therefore traditional Divisia construction gives them a higher weight. These liquid assets might be expected, according to macroeconomic theory, to be the most important assets in affecting inflation if the supposed relationship between monetary services and inflation holds true. Yet it is evident from Table 1 that the less liquid interest-bearing deposits are more important in our attempt to model inflation based on Divisia components. If our results turn out to have general applicability, they suggest that macroeconomic theory requires amendment.

Inflation 'forecasting' results from the three networks are shown in Table 2 and figure 2. 'Divisia' refers to traditional Divisia rather than the new index derived from our experiment.

Network	*Variables*	*Train RMSE*	*Test RMSE*
Components	N/C, NIBD, IBSD, IBTD, BSD, Π_{t-1},T	0.0030129	0.0073013
New Divisia	New index, Π_{t-1},T	0.0068229	0.0091321
Divisia	Divisia, Π_{t-1},T	0.0056577	0.0232522

Table 2: Error for the three trained networks

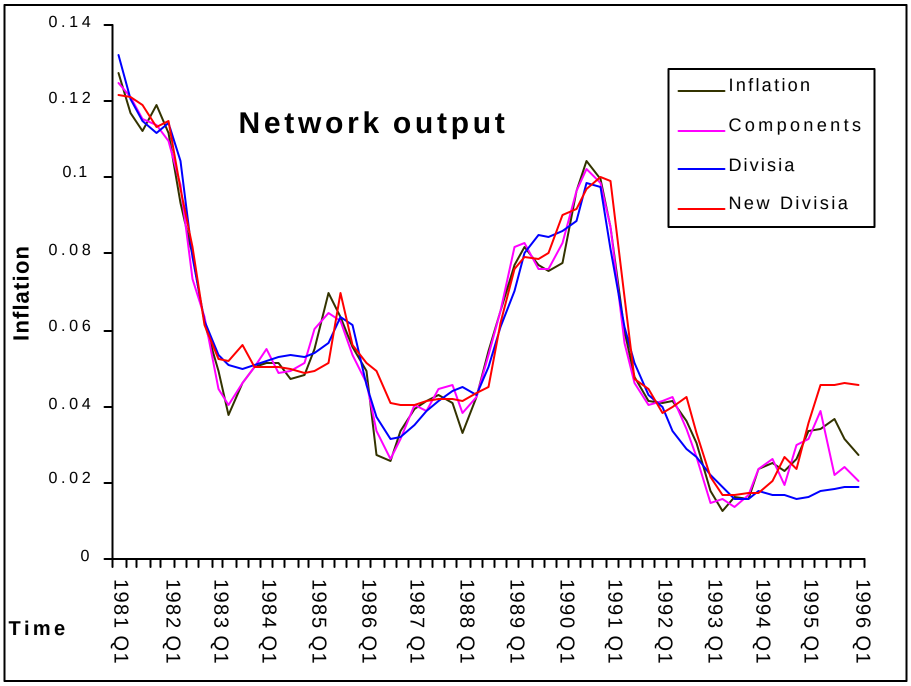

Figure 2: Output for the three networks compared to actual inflation

These results show that the RMS error is lower when the network is trained on the components of Divisia, rather than on the aggregate, both within-sample and out-of-sample. Moreover, when a new aggregate is constructed which employs weights derived from this first network, it holds up well against the traditional Divisia and even out-performs it on the out-of-sample results.

Is it surprising that the 'raw' components appear to lead to a better model than either weighted index? No, because neural networks themselves have the ability to model relationships far more complex than most indices can manage to embody. This network had the ability to take account of complex interactions *between* variables. Moreover, the sensitivities derived from this network and applied as our 'new' index weights are average sensitivities, and in our index the weights are applied equally for every quarter in the series over a period of fifteen years.

What is more surprising is that our crude 'new' index should rival the traditional Divisia index, which has a complex construction to take account of economy sectors, quarterly changes in interest rates and personal and corporate taxation. Future development of this research will permit further experimentation. The neural network technique, which has the ability to approximate any unknown mapping, provides researchers working in the field of economics with a powerful new tool that may enable them to better understand the interrelationships of explanatory variables.

Concluding remarks

Results presented in this paper are generated by use of a powerful tool that allows greater flexibility of functional form than that currently obtainable by use of conventional econometric methodology. The combination of Divisia measures of money with the artificial neural network offers a promising starting point for the development of an improved model of inflation. The role of monetary aggregates in the major economies today has largely been relegated to one of a leading indicator of economic activity, along with a range of other macroeconomic variables. However, further empirical work on Divisia money may serve to restore confidence in former well established money-inflation links and re-establish money as an effective macroeconomic policy tool in its own right. In any case, the results presented in this paper appear to open up a new area for debate. This application of the neural network methodology to examine the money - inflation link is highly experimental in nature and in keeping with the pioneering work conducted by the current authors for the UK, USA and Italy (Gazely and Binner, 2000) the overriding feature of this research is very much one of simplicity. It is virtually certain in this context that more accurate inflation forecasting models could be achieved with the inclusion of additional explanatory variables, particularly those currently used by monetary authorities around the world as leading indicator components of inflation.

References

Barnett WA, (1980) 'Economic monetary aggregates; An application of index number and aggregation theory', *Journal of Econometrics,* **14,** [1] 11-48.

Barnett, WA, Fisher, D and Serlitis, A (1992) Consumer theory and the demand for money. *Journal of Economic Literature, Vol. XXX* (December), *2086-119*.

Barnett, WA, Offenbacher, E and Spindt, P (1984) The new Divisia monetary aggregates, *Journal of Political Economy,* December, 1049-85.

Barnett, WA and Spindt, P (1979) The velocity behaviour and informational content of Divisia monetary aggregates. *Economic Letters,* Vol. 4, p51-7.

Belongia, MT and Chalfant, JA (1989) The changing empirical definition of money: some estimates from a model of the demand for money substitutes. *Journal of Political Economy 97 387-97.*

Binner, JM (1990) The construction, interpretation and analysis of Divisia monetary aggregates for the UK. Unpublished Ph.D. thesis. *Department of Economics, University of Leeds.*

Chrystal, KA and MacDonald, R (1994) Empirical evidence on the recent behaviour and usefulness of simple sum and weighted measures of the money stock. *Federal Reserve Bank of St. Louis Review,* March/April, p 73-109.

Divisia F, (1925) 'L'indice monétaire et al théorie de la monnaie', *Review d'Economy Politique,* **39** 980-1008.

Dorsey, R (1998) Neural networks with Divisia money: Better forecasts of future inflation? in Belongia, MT and Binner, JM (eds.) *Divisia Monetary Aggregates; Theory and Practice?* MacMillan Press, Basingstoke, UK.

Drake L, Chrystal KA and Binner JM, (2000) 'Weighted monetary aggregates for the UK', in Belongia, MT and Binner, JM (eds.) *Divisia Monetary Aggregates; Theory and Practice?* MacMillan Press, Basingstoke, UK.

Drake L and Mullineux AW, (1997) 'One Divisia money for Europe', *Applied Economics* **29**, 775 – 786.

Ford, JL and Mullineux, AW (1996) Financial innovation and monetary aggregates in the UK, 1977-93. In *Financial Innovation, Banking and Monetary Aggregates*, ed. A. Mullineux, Edward Elgar, Cheltenham, UK, *13-38*.

Gazely AM and Binner JM, (2000) 'The application of neural networks to the Divisia index debate: Evidence from three countries', *Applied Economics* **32,** forthcoming, September.

Mullineux, AW (1996) *Financial Innovation, Banking and Monetary Aggregates*. (Ed.) Edward Elgar, Cheltenham, UK. Chapter 1.

Serletis A (1988) The empirical relationship between money, prices and income revisited, *Journal of Business and Economic Statistics*, July, 351-358.