

**ENGR 240 Science of Materials**  
Homework 6 Solutions – Electrical Properties, Part 1  
14 points

1. [1 point] Callister Problem 19.3 – conductivity/resistivity calculations.

19.3 This problem asks that we compute, for an aluminum wire 4 mm in diameter, the maximum length such that the resistance will not exceed 2.5 W. From Table 19.1, for aluminum,  $S = 3.8 \times 10^7 \text{ (W}\cdot\text{m)}^{-1}$ . If  $d$  is the diameter then, combining Equations (19.2) and (19.4) leads to

$$l = R_s A = R_{sp} \left( \frac{d}{2} \right)^2$$
$$= (2.5 \text{ W}) [3.8 \times 10^7 \text{ (W}\cdot\text{m)}^{-1}] (\pi) \left( \frac{4 \times 10^{-3} \text{ m}}{2} \right)^2 = 1195 \text{ m}$$

2. [2 points] The Travltoast toaster by Empire (1970's) was designed to work with the 12 V adapter in your car – great for breakfast on the go! The heating element wires in the Travltoast are made from a material called Nichrome, which is a common electrical resistance alloy that has a composition of approximately 80 wt.% nickel and 20 wt.% chromium. The Nichrome heating wires are 0.5 mm diameter Nichrome wire ( $\sigma$  for Nichrome is  $9.3 \times 10^5 \text{ (}\Omega\cdot\text{m)}^{-1}$ ), and the total length of the heating element wires is 4 meters.

- a. Calculate the current flow through the wires.
- b. The life of toaster heating elements increases if they are made from larger diameter wire. If the diameter of the heating element wires were increased from 0.5 mm to 2 mm, what would happen to the resistivity of the Nichrome wires?

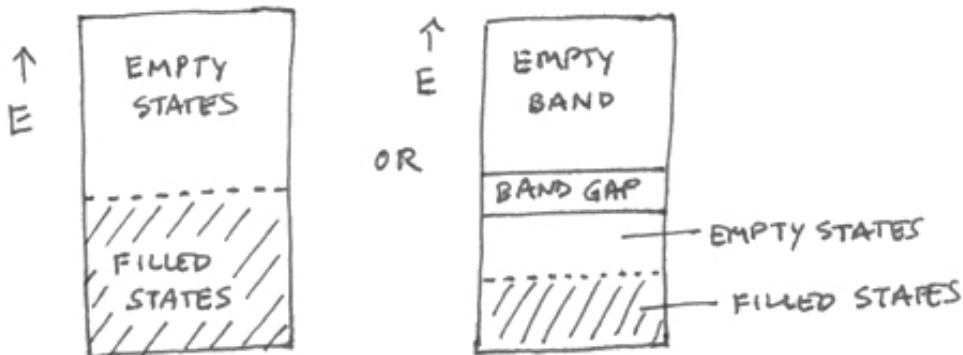
a. Current is calculated using Equations 19.3 and 19.4 as follows:

$$I = \frac{VA}{\rho l} = \frac{\sigma VA}{l} = \frac{(9.3 \times 10^5 \text{ }\Omega\text{m})(12V) \left( \frac{\pi}{4} (0.0005\text{m})^2 \right)}{4\text{m}} = 0.55\text{A}$$

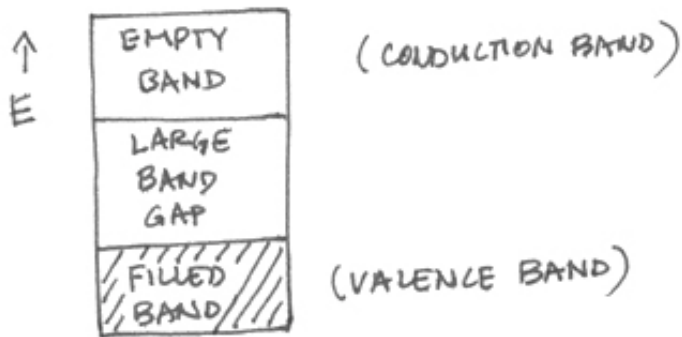
- b. Resistivity is a material property, and it does not change as a function of geometry. So, if we look at the two different sizes of Nichrome wires at a constant temperature, their resistance will be different, but the resistivity of the wires will be the same.

3. [3 points] Draw a schematic energy band diagram for (i) nickel, (ii) aluminum oxide, and (iii) germanium. Be sure to label all the important features on your diagrams.

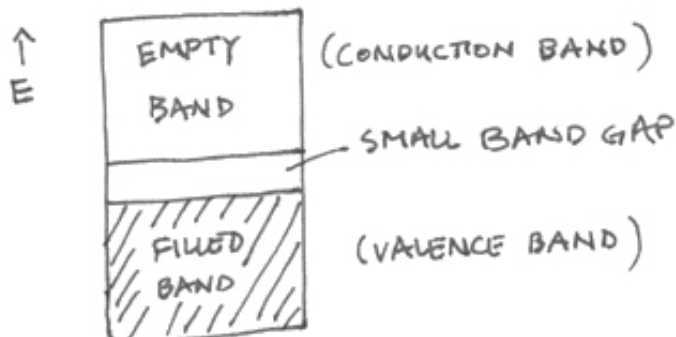
(i) Ni - PARTIALLY-FILLED d BAND; METAL; CONDUCTOR



(ii) ALUMINUM OXIDE,  $Al_2O_3$  - INSULATOR



(iii) GERMANIUM - SEMICONDUCTOR



4. [2 points] Callister Problem 19.13 – electron mobility.

19.13 (a) The number of free electrons per cubic meter for copper at room temperature may be computed using Equation (19.8) as

$$\begin{aligned}n &= \frac{s}{A_e A m_e} \\&= \frac{6.0 \times 10^7 (\text{W}\cdot\text{m})^{-1}}{(1.602 \times 10^{-19} \text{ C})(0.0030 \text{ m}^2/\text{V}\cdot\text{s})} \\&= 1.25 \times 10^{29} \text{ m}^{-3}\end{aligned}$$

(b) In order to calculate the number of free electrons per copper atom, we must first determine the number of copper atoms per cubic meter,  $N_{\text{Cu}}$ . From Equation (4.2)

$$\begin{aligned}N_{\text{Cu}} &= \frac{N_A r}{A_{\text{Cu}}} \\&= \frac{(6.023 \times 10^{23} \text{ atoms/mol})(8.94 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)}{63.55 \text{ g/mol}} \\&= 8.47 \times 10^{28} \text{ m}^{-3}\end{aligned}$$

The number of free electrons per copper atom is just

$$\frac{n}{N} = \frac{1.25 \times 10^{29} \text{ m}^{-3}}{8.47 \times 10^{28} \text{ m}^{-3}} = 1.48$$

5. [4 points] At room temperature, copper-tungsten alloys of any composition will exist as two separate solid phases. Molybdenum and tungsten, on the other hand, are completely soluble in the solid state at room temperature (they form a single phase solid). The conductivity values of tungsten, molybdenum, and copper are as follows:

Copper:  $\sigma = 5.8 \times 10^7 \text{ } (\Omega\text{-m})^{-1}$

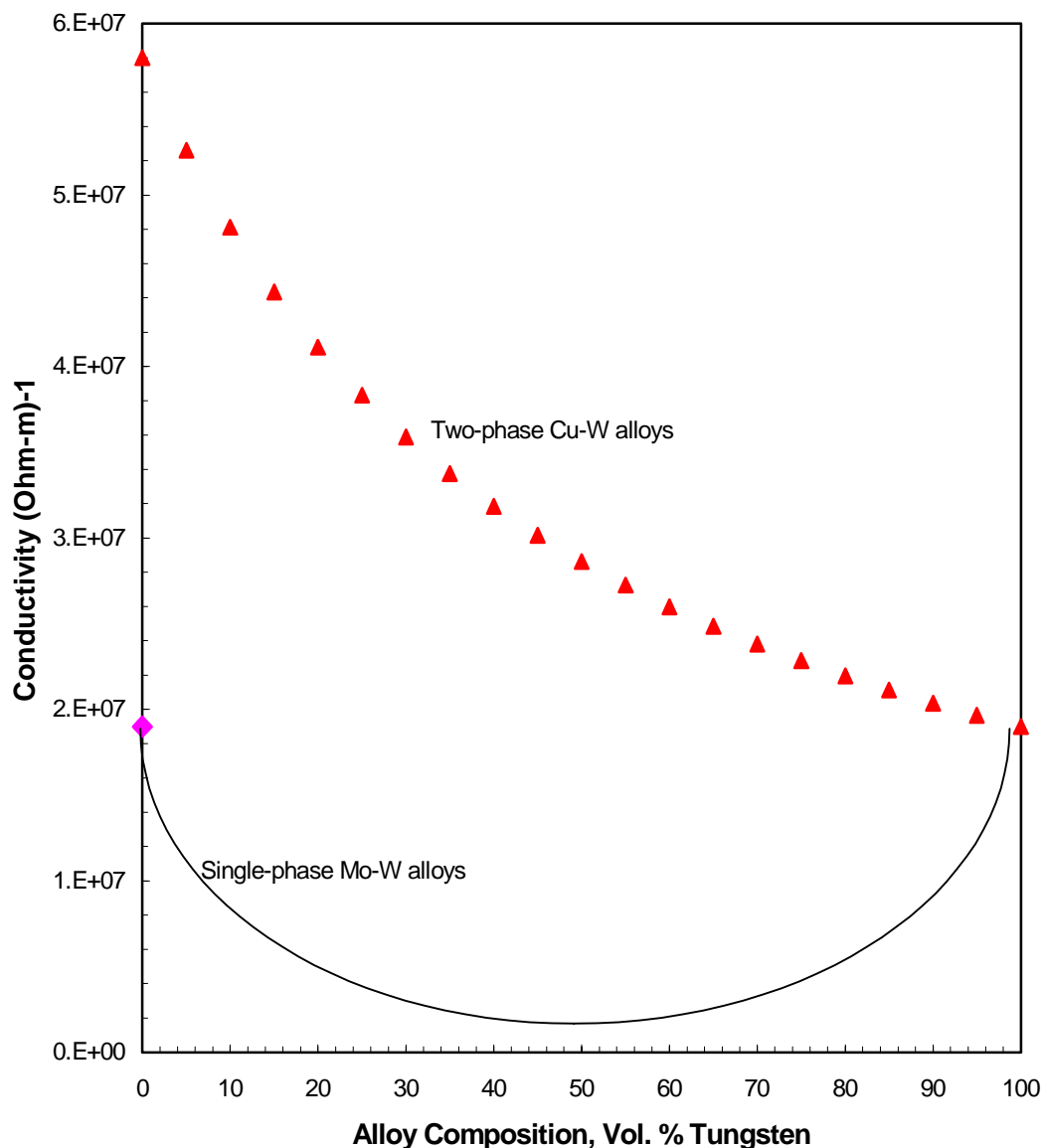
Tungsten:  $\sigma = 1.9 \times 10^7 \text{ } (\Omega\text{-m})^{-1}$

Molybdenum:  $\sigma = 1.9 \times 10^7 \text{ } (\Omega\text{-m})^{-1}$

- a. Use Excel or another computer graphing program to plot the conductivity of tungsten-copper alloys as a function of volume percent tungsten. Compute and plot conductivity values at every 5 volume % tungsten increment.
- b. On the same graph, schematically plot the conductivity of single-phase molybdenum-tungsten alloys as a function of volume percent tungsten.

- a. Use Equations 19.12 and 19.4 to calculate conductivity values. Since I asked you to calculate the conductivity of 2-phase alloys, not the resistivity, the curve for the Cu-W two-phase alloys is not linear.
- b. Use Equation 19.11 to estimate the shape of the Mo-W single phase alloy conductivity curve.

**Conductivity of Cu-W and Mo-W Alloys**



6. [2 points] Using data in Table 19.2, estimate the conductivity of InSb at a temperature of 200 °C.

First, use Equation 19.18 and the information in Table 19.2 to solve for the constant C at room temperature:

$$\ln \sigma = C - \frac{E_g}{2kT}$$

The constant C at 25 °C is equal to 13.21.

Use your calculated C and the given activation energy to calculate the conductivity at 200 °C:

$$\ln \sigma = 13.21 - \frac{0.17eV}{2(8.62 \times 10^{-5} \frac{eV}{atomK})(473K)} = 6.8 \times 10^4 (\Omega \cdot m)^{-1}$$